

DOCUMENT RESUME

ED 185 166

UD 020 360

AUTHOR Suydam, Marilyn N.
 TITLE What Do We Know About Teaching and Learning in Urban Schools? Volume 11: Mathematics and the Urban Child: A Review of Research.
 INSTITUTION Central Midwestern Regional Educational Lab., Inc., St. Louis, Mo.
 SPONS AGENCY National Inst. of Education (DHEW), Washington, D.C.
 PUB DATE 79
 NOTE 127p.; Paper prepared for the Urban Education Program, CEMREL, Inc.'s National Conference on Urban Education (St. Louis, MO, July 10-14, 1978). For related documents see UD 020 351-361 and UD 020 363.
 EDRS PRICE MF01/PC06 Plus Postage.
 DESCRIPTORS Academic Achievement: Elementary Secondary Education; Literature Reviews; *Mathematics Curriculum; *Mathematics Education; Socioeconomic Status; *Student Characteristics; *Teaching Methods; *Urban Schools; *Urban Teaching

ABSTRACT

Some of the factors that have been considered in relation to the teaching of mathematics, especially to students in urban schools, are examined in this paper. First, information on the mathematics achievement levels of students with different demographic characteristics is presented: the extreme variance in achievement between students from high socioeconomic urban areas and those from low socioeconomic urban areas is shown. An extensive review of existing practices in mathematics education (in both urban and non urban schools) follows. School and classroom organizational patterns, curricula and instructional programs, class size, time allotment and use, teaching approaches, individualized instruction, student characteristics, instructional materials, the impact of Federal funding, and progress assessment (evaluation) are covered in this discussion. Next, strategies pertaining specifically to urban students, as identified in an examination of relevant ERIC materials, are outlined. Finally, long term research priorities are identified, with an emphasis on the scope of mathematics education, the teaching methods to be employed, and the effects of particular instructional procedures on urban children. An extensive bibliography is attached. (GC)

 * Reproductions supplied by EDRS are the best that can be made *
 * from the original document. *

ED185156

WHAT DO WE KNOW ABOUT TEACHING AND LEARNING IN URBAN SCHOOLS?

U.S. DEPARTMENT OF HEALTH,
EDUCATION & WELFARE
NATIONAL INSTITUTE OF
EDUCATION

THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY.

Volume 11

Mathematics and the Urban Child: A Review of Research

Marilyn N. Suydam
The Ohio State University

Paper prepared for the National Conference on Urban Education,
held in St. Louis, Missouri, July 10-14, 1978.
Conducted by the Urban Education Program, CEMREL, Inc.,
with support from the National Institute of Education.



CEMREL, Inc., St. Louis, Missouri

FEB 26 1980

Prepared by CEMREL, Inc., a private nonprofit corporation supported in part as an educational laboratory by funds from the National Institute of Education, Department of Health, Education and Welfare. The opinions expressed in this publication do not necessarily reflect the position or policy of the National Institute of Education, and no official endorsement should be inferred.

Copyright on this document is claimed only during the period of development, test, and evaluation, unless additional authorization is granted by the National Institute of Education to claim copyright on the final version. For information on the status of the copyright claim, contact either the copyright proprietor or the National Institute of Education.

Copyright 1979, CEMREL, Inc.

Library of Congress catalog card
number 79-54737
Manufactured in the
United States of America

CEMREL, Inc.
3120 59th Street
St. Louis, MO 63139

Table of Contents

ACKNOWLEDGMENTS.....	vii
FOREWORD.....	xiii
MATHEMATICS AND THE URBAN CHILD: A REVIEW OF RESEARCH.....	1
PREFACE.....	1
INTRODUCTION.....	4
I. THE STATUS OF ACHIEVEMENT OF STUDENTS IN URBAN SCHOOLS.....	5
II. EVIDENCE ON EXISTING PRACTICES IN MATHEMATICS INSTRUCTION.....	12
III. EFFECTIVE STRATEGIES FOR URBAN STUDENTS.....	67
IV. SYNTHESIS OF EVIDENCE.....	73
V. RESEARCH PRIORITIES.....	83
VI. RECOMMENDATIONS.....	88
REFERENCES.....	91

Acknowledgments

The Urban Education Program, CEMREL, Inc., would like to thank the National Institute of Education and the Regional Joint Planning Group of the Urban Education Program for its support and assistance in planning the conference.

Conference Chairperson

Harriet Doss Willis
Director
Urban Education Program

Conference Participants

Nettie S. Ammer
Research Associate
St. Louis University

Eva Baker
Director
Center for the Study of Evaluation
UCLA Graduate School of Education

Carrie E. Bash
Division of Community Programs
Urban League of St. Louis

Isabel Beck
Co-director
Language & Communications Unit
Learning Research and Development Center
University of Pittsburgh

Janet Cheatham Bell
Urban/Ethnic Studies Coordinator
Indiana Department of Public Instruction

David Bennett
Deputy Superintendent
Milwaukee Public Schools

Edward Binkley
Director of Research & Evaluation
Metro Schools
Nashville, Tennessee

Milton Bins
Senior Associate
Policy Development
The Council of Great City Schools
Washington, D.C.

Dean Bowles
Director
Home-School-Community Project
Wisconsin Research and Development Center

Ronald A. Boyd
Associate Superintendent
Department of Education
State of Indiana

Emerson Brown
Board of Directors
CEMREL, Inc.

L.D. Buford
Principal
St. Louis Public Schools

William D. Callian, Jr.
Assistant Superintendent
Curriculum Instruction
Memphis City Schools

Angeline P. Caruso
Associate Superintendent
Department of Curriculum &
Instructional Services
Board of Education
City of Chicago

Courtney Cazden
School of Education
Harvard University

Ned Chalker
Teaching & Learning Program
National Institute of Education

Francis S. Chase
Director of Urban Education
Dallas, Texas

Rosemary Christensen
Director of Indian Education
Minneapolis Public Schools

Doris J. Clemons
Region Five Office
Detroit Board of Education
Detroit, Michigan

Frances Coe
Board of Directors
CEMREL, Inc.

David Colton
Graduate Institute of Education
Washington University
St. Louis, Missouri

Lloyd M. Cooke
Vice Chairman
Economic Development Council of
New York City

Dallas Daniels
Director
Equal Employment Opportunity
Indianapolis Department of Public
Instruction

A. W. Dirks
Director
Research, Planning & Development
Wichita Public Schools

Lorenza Dixon
Curriculum Coordinator
Indianapolis Public Schools

John W. Dobbs
Assistant Superintendent
School & Community Affairs
Department of Education
State of Michigan

Domingo Dominguez
Director
Bilingual & Migrant Program
Southwest Educational Development
Laboratory

Ronald Edmonds
Director
Center for Urban Studies
Graduate School of Education
Harvard University

Nancy Evers
Department of Educational
Administration
University of Cincinnati

Joseph L. Felix
Director
Program Evaluation
Cincinnati Public Schools

Marian Kilbane-Flash
Supervisor
Research & Development
Cleveland Public Schools

Anita Ford
Co-chairman
Citizens Education Task Force
St. Louis, Missouri

Ruges R. Freeman
Coordinator
Secondary Student Training
Southern Illinois University
Edwardsville, Illinois

Gary Gappert
New Jersey Department of Education

Booker Gardner
Wisconsin Research & Development
Center for Individualized Schooling
Madison, Wisconsin

Howard Gholson
Policy Analysis
Title I
U.S. Office of Education

Henry Givens
Assistant Commissioner
Department of Elementary &
Secondary Education
State of Missouri

Edmund Gordon
Director
Institute for Urban &
Minorities Education
Teachers College
Columbia University

Milton Goldberg
Educational Organizations &
Institutions
National Institute of Education

John H. Grate
Director
Planning & Development
Cincinnati Public Schools

Eldon Grossner
Manager
LEA Services
Illinois Office of Education

Rex Hagans
Northwest Regional Educational
Laboratory

Robert J. Havighurst
Department of Education
The University of Chicago

Garland M. Hawkins
Director
Music Education
Curriculum Center School Dist. 189
East St. Louis, Illinois

Paul Hill
The Rand Corporation

Keith Hyde
Planning & Development
Des Moines Public Schools

John Ingram
Wisconsin Research & Development
Center for Individualized Schooling

Barbara Jackson
Associate Dean
Department of Administration &
Supervision
School of Education
Atlanta University

James N. Jacobs
Superintendent
Cincinnati Public Schools

Sylvia K. Jones
State & Federal Grants
Toledo Public Schools

Preston C. Kronkosky
Field Services & Dissemination
Southwest Educational Development
Laboratory

Daniel U. Levine
Director
Center for the Study of
Metropolitan Problems
in Education
University of Missouri-Kansas City

Joseph D. Liggins
Assistant Superintendent for Staff
Development
Houston Independent School District

Reece Little
Director of Curriculum Development
Jefferson County Public Schools
Louisville, Kentucky

Irene Lober
Superintendent
University City Public Schools
St. Louis, Missouri

William Lobosco
ESEA Title I
U.S. Office of Education

Catherine Lyon
Center for the Study of Evaluation
UCLA Graduate School of Education

Mary Meehan
District Coordinator
The School District of Kansas City

LaMar Miller
Director of Secondary Education
New York University

Len Nachman
Division of Planning & Development
Minnesota Department of Education

Tom Odneal
Director
Title IV
Missouri Department of Education

Joan Orender
Project Coordinator for
Distribution of Information
Nebraska Department of Education

George A. Parry
George A. Parry & Associates
River Forest, Illinois

William Phillips
Minneapolis Public Schools

Andrew C. Porter
Institute for Research on Teaching
College of Education
Michigan State University

Electra Price
Oakland Unified School District
Oakland, California

Walter E. Richardson
Supervisor
Compensatory Instruction
Columbus Public Schools

J. Robert Sampson
Assistant Manager
Program Planning & Development
Illinois Office of Education

Judah L. Schwartz
Professor of Engineering, Science
and Education
Massachusetts Institute of
Technology

Cleona Shortridge
St. Louis Public Schools

Barbara Shull
Conference on Education
St. Louis, Missouri

Edward E. Smith
Right to Read Effort
Washington, D.C.

Douglas Smith
Administrative Assistant to
the Governor
State of Michigan

Frank Sobol
Basic Skills Group
National Institute of Education

George Spicely
U.S. Office of Education

Jane Stallings
Stanford Research Institute

Tom Stefonek
 Director of Planning & Evaluation
 Research
 Wisconsin Department of Public
 Instruction

Vera John-Steiner
 Department of Educational
 Foundations
 College of Education
 The University of New Mexico

Ronald Stodghill
 Associate Superintendent
 St. Louis Public Schools

Marilyn Suydam
 ERIC Information Analysis Center
 for Science, Math &
 Environmental Education
 The Ohio State University

Maud Thomas
 U.S. Office of Education
 Kansas City Regional Office

Concepcion Valadez
 Graduate School of Education
 University of California
 Los Angeles

Mamie Wandick
 Program Officer
 U.S. Office of Education
 Kansas City Regional Office

Phyllis Weaver
 School of Education
 Harvard University

Billy-Belle Weber
 Weber, Fuhrmann & Associates
 East St. Louis, Illinois

Joyce Weddington
 Director
 Research and Evaluation
 Memphis City Schools

Rachel T. Weddington
 University Dean for Teacher
 Education
 The City University of New York

Lores Wells
 Harris Teachers College
 St. Louis, Missouri

Robert Wentz
 Superintendent
 St. Louis Public Schools

Barbara Woods
 Afro-American Studies
 St. Louis University

Saul Yanofsky
 National Institute of Education

Savannah M. Young
 St. Louis Public Schools

Allen F. Zondlak
 Director of Planning
 Detroit Public Schools

URBAN EDUCATION PROGRAM STAFF

John Bass
Resource Coordinator

Alfreda Brown
Resource Specialist

Pamelia Coe
Research Associate

Eileen Hamilton
Secretary

Nellie H. Harrison
Dissemination-Curriculum Specialist

Donald Miller
Research Advisor

Harriet Doss Willis
Director

Astacia Wright
Research Assistant

STAFF FOR WHAT DO WE KNOW ABOUT
TEACHING AND LEARNING IN URBAN
SCHOOLS?

Harriet Doss Willis
Editor

Margaret Solomon
Manuscript Editor
Production Coordinator

Linda Cambell
Office Manager

Marlene Dandridge
Floor Clerk

Arneita Gray
Secretary

Angela Reed
Secretary

Dian Taylor
Secretary
Office Manager

Karen Temmen
Secretary

Foreword

This monograph is one of a series of papers on the issues of instruction and learning in urban schools that were presented at a conference held July 10-14, 1978, in St. Louis, Missouri, sponsored by the Urban Education Program, CEMREL, Inc., and supported by the National Institute of Education. It is our hope that both the conference and the series of fourteen monographs that resulted from it will assist educational researchers and school practitioners in identifying and analyzing instructional and learning problems in urban school settings, and will contribute to the development of strategies for the improvement of schooling for students.

Titled "What Do We Know About Teaching and Learning in Urban Schools?", the 1978 conference focused on an examination of research findings on the teaching-learning process in areas most germane to and under control of the schools. The key problems to which the papers are addressed are the assessment of learning outcomes and the analysis of the relationships between instructional and other inputs and learning outcomes. Important contextual issues not under the full control of schools, such as community and parental involvement and the impact of federal and state governments in improving educational opportunities, were addressed at the conference as well.

The conference was conceived, in part, as an initial step in the development of a state-of-the-art review of the most critical issues faced by educators in urban schools who struggle with the morass of general educational problems, exacerbated by decreasing public confidence in large city schools, declining student enrollment, and increasing numbers of poor and minority students. While the conference does not exhaust discussion of the problems

that are particularly significant in the urban school setting, it forms a convenient focus for viewing the larger issues affecting education in our cities.

The educational innovation and change growing out of more than ten years of effort since the passage of the Elementary and Secondary Education Act of 1965 -- the primary stimulus for federal support of research and development in education -- has had the least impact on the student populations typical of the urban school setting. With this concern as a primary consideration, ten months prior to convening the conference, CEMREL's Urban Education Program had begun work with a network of twenty big-city school districts and state departments of education in Illinois, Indiana, Iowa, Kansas, Kentucky, Michigan, Minnesota, Missouri, Nebraska, Ohio, Tennessee, and Wisconsin. Thus, the conference and the research findings of national authorities was undertaken to lay a foundation for future work in urban educational research and development and to undergird present efforts at improvement of schools in the region.

The Conference Design

To guide the development of the presenters' preparation of their papers, topical questions were prepared for them by Urban Education Program Staff and a paper format was recommended. Each presenter prepared a full paper based on the topical questions and results were summarized by the writers during general sessions of the conference. Because of a wide range of perspectives held on the conference issues, discussions precipitated by reactor panels composed of other prominent researchers, practitioners, and community leaders in education followed each general conference session. More than one hundred educational researchers and practitioners from across the country attended the conference. The invited participants came from school districts and state departments of education from nineteen states; thirteen institutions of higher education; four federal agencies; numerous educational laboratories and research and development centers; and seven social service agencies.

The conference was divided into two types of sessions: general sessions, wherein the nationally recognized speakers who wrote papers presented their summaries; and forum sessions, wherein reactors helped analyze the presentations and examine the concrete recommendations put forth by the general-session speakers. All participants were encouraged to present questions and reactions during these forum sessions.

Francis S. Chase, Director of Urban Education Studies, Dallas, Texas, gave the keynote address at the opening session of the conference, a banquet on Monday evening, July 10. His topic, "Promising Developments in Urban Education," summarized field studies that he was conducting under sponsorship of the Council of Great Cities Schools and the University Council for Educational Administration under a grant from the Spencer Foundation. Chase is Professor Emeritus of the University of Chicago, where he also served as Chairman and Dean of Education for ten years.

The Conference Topics

Each of the presenters had been asked to design a paper by responding to salient topical questions, drawn up by CEMREL's Urban Education Staff. The questions that were posed originally for each of the writers are presented below. Most of the resulting papers were developed into monographs for this series.

Compensatory Education: A
Congressional Perspective
Christopher T. Cross
Minority Staff Director
Committee on Education and
Labor
U.S. House of
Representatives.

What is the current state of the evidence concerning projected changes in legislative mandates, and what may we expect related to changes in policy governing federally funded compensatory education programs?

Effective Teacher-Support
Systems to Improve
Instructional Conditions in
Schools

Joseph D. Liggins
Assistant Superintendent
for Staff Development
Houston Independent School
District

Assessment That Respects
Complexity in Individuals
and Programs

Judah L. Schwartz
Professor of Engineering,
Science and Education
Massachusetts Institute of
Technology

Achievement Tests in Urban
Schools: New Numbers

Eva Baker
Director
Center for the Study of
Evaluation
University of California,
Los Angeles

Summer Drop-off and The
Effectiveness of
Compensatory Instruction

Paul T. Hill
The Rand Corporation

What teacher support systems are most effective in improving instructional conditions for students? What is the role of in-service training, and what types of in-service training can be recommended to urban educators? What other resources--such as additional curricular materials, support from teacher aides and volunteers, time re-allocations, team teaching arrangements, etc.--can be demonstrated to be effective in improving students' learning opportunities? What is the state of the evidence?

How can we best measure and analyze the effects of instruction on students' learning in the major school subjects? What are the most appropriate times and strategies for measuring the effects of schooling on student learning? What do we learn from means of assessment other than standardized achievement testing? When and under what circumstances is it most appropriate to use alternative or complementary strategies for assessing program effectiveness?

How can we best measure and analyze the effects of instruction on students' learning in the major school subjects? What are the most appropriate times to measure the effects of schooling on student learning? What are the cautions which should be taken into account before any attempt to evaluate the effectiveness of programs primarily on the basis on standardized achievement data, especially when the number of individual implementations is large and the actual conditions under which the implementations took place are not well documented?

What has the era of compensatory education program development contributed to knowledge about achievement in the basic school subjects in urban schools? Where is the evidence? What, if anything, does the evidence indicate about methods for improving basic skills achievement? What, if any, recommendations can be made to school district planners on the basis of existing evidence?

Instructional Ingredients
for the Development of
Beginning Reading

Competence

Isabel Beck

Co-Director

Learning Research and
Development Center

University of Pittsburgh

Much research, program development, and implementation of efforts to improve achievement in basic school subjects has been concentrated in the area of reading. What approaches to instruction in reading have been successful in increasing achievement? Is there evidence that certain approaches are best suited to the needs of particular populations? Do these effects generalize to other areas of school learning, such as mathematics, social studies, and science?

A Discussion of The
Literature and Issues
Related to Effective
Schooling

Ronald Edmonds

Director

Center for Urban Studies

Graduate School of

Education

Harvard University

Are there schools that have demonstrated a higher degree of effectiveness in the delivery of instruction in the basic school subjects than others serving similar student populations (in terms of socioeconomic backgrounds, family characteristics, minority populations, etc.)? What characteristics distinguish these schools from less successful schools in similar situations? What is the evidence that the explanatory characteristics have been correctly identified, and how strong is that evidence?

The Educational Promise of
Cultural Pluralism

Vera John-Steiner and

Larry Smith

Department of Educational
Foundations

College of Education

The University of New Mexico

How can we approach problems of teaching and learning in urban schools so that the racial and cultural diversity of the school population is recognized, respected, and utilized constructively, to the extent possible? How do we differentiate between basic academic skills, which all students need to acquire for successful participation in our heterogeneous society, and curriculum decisions which can and should be tailored to the needs and interests of different sub-cultural groups? Do students benefit from participation in racially and culturally heterogeneous classrooms, or perhaps from certain kinds of heterogeneous instructional settings but not others? What is known about these issues, and how much consensus is there about the cultural values on which schooling must be based?

New Perspectives on School
District Research and
Evaluation

Catherine Lyon
Center for the Study of
Evaluation
University of California,
Los Angeles

Effective Teaching and
Learning in Urban Schools

Jane A. Stallings and
Shirley W. Hentzell
Stanford Research Institute

Mathematics and The Urban
Child

Marillynn Suydam
Center for Science,
Mathematics, and
Environmental Education
The Ohio State University

Basic Skills in Urban
Schools: A View From The
Bilingual Classroom

Concepcion Valadez
Graduate School of
Education
University of California,
Los Angeles

What is the role of research and development processes in improvement of instruction and learning in basic school subjects in large urban systems? In what form are these processes being applied to the teaching and learning process in urban schools? What is the evidence and how solid are the indications that use of these processes has resulted in improved student achievement?

Looking specifically at the interaction of student, teacher, curricular and school organization characteristics, what instructional conditions encourage achievement in basic school subjects? What evidence is there, and how good is it? How are the effects of instructional conditions on student achievement measured? On the basis of existing evidence and trends, what recommendations can be made for improving students' instructional environments?

Mathematics is a school subject which is generally considered basic, insofar as mathematical skills are required for successful adult functioning in our complex society. What evidence is there to indicate which approaches to instruction in mathematics have been successful in increasing achievement? Is there evidence that certain approaches are best suited to the needs of particular populations? Do these effects generalize to other areas of school learning, such as reading, social studies, and science?

Since the second largest minority population served by many urban school districts is a bilingual population, what is the current state of the evidence about the effects of programs developed especially for bilingual students? What, if anything, does the evidence indicate about the effects of such programs on basic skills achievement?

School-Community Relations
and The Urban School

Dean Bowles

Director

Home-School-Community
Project

Wisconsin Research and
Development Center for
Cognitive Learning

What do we know or what can we learn about the influence of parental and community involvement in schooling on student achievement? How does parental and community involvement affect the teaching-learning process? If information is not available to answer these questions on even a tentative basis, what types of research are recommended to study this aspect of students' lives more effectively?

Frank Sobol

Basic Skills Group

National Institute of
Education

Are there research and development products which have contributed to improved instruction and increased student achievement in basic school subjects in large urban school systems? What is the evident and how solid are the indications that use of these products has resulted in improved student achievement?

A Summary of The Conference
Findings

Edmund W. Gordon

Director

Institute for Urban and
Minority Education
Teachers College
Columbia University

In light of the evidence produced by examination of these issues, what concrete recommendations can be made to school districts concerning the improvement of instruction and learning in basic school subjects? In light of the same evidence what research and development agenda should be the focus of future study and program development?

Dr. Francis S. Chase,
Director of Urban Education
Dallas, Texas

Keynote Speaker for Conference Banquet:
"Promising Developments in Urban Education"

--Harriet Doss Willis
Director
Urban Education Program

Mathematics and the Urban Child: A Review of Research

Marilyn N. Suydam
The Ohio State University

PREFACE

The following is a bulletin from the National Council of Teachers of Mathematics, dated January 1978. Its focus seems particularly relevant to this paper.

Teaching Mathematics in the Urban School

Some recent headlines: "The ABCs of School Violence"; "In the Ghetto: Dwindling Hopes"; "Bribes for Truants: Officials Fed up with Absenteeism"; "High Schools in Trouble: A Tale of Three Cities"; "SAT Scores Decline for Fourteenth Year."

Every member of the National Council of Teachers of Mathematics should be deeply concerned about the social climate in urban schools. Nowhere are problems more immediate, more critical, or more difficult. We, the NCTM Board of Directors, hereby state our support for, and our commitment to, our colleagues who are addressing these

MARILYN N. SUYDAM is Professor of Mathematics Education at The Ohio State University.

problems in positive ways: those teachers who work at communicating mathematics in city classrooms, those supervisors and administrators who actively support the instructional program, and those teacher trainers who give special attention to the particular problems urban teachers face.

Headlines present symptoms; we of the education community, however, must seek to confront more basic issues. Some of these--the breakdown of the home unit, drug use, even generalized anger and frustration--are factors over which the schools have little control; but the failure of many students to achieve even minimum skills, along with their subsequent frustration, is a problem that the schools must address. Our concern as a mathematics education community must be at once general, for the improvement of our students' ability to cope with society, and specific, for the improvement of their ability to deal with mathematical problems. These goals apply to all students in all types of communities. We fail to achieve them with some students in all settings, but we fail to achieve them most often in our cities. The problems of urban education should be recognized as a priority that demands a major commitment of resources.

Teachers have a right to expect certain forms of support. First, society owes to every classroom teacher, among other things, a proper instructional setting, security, a sense of worth, and students to teach. Can teachers be expected to teach when threatened and thwarted? When they do not have adequate facilities and necessary materials? Can they be expected to teach students who do not attend class? We call for a response to these problems by administrators, school boards, teacher organizations, and parents. Specifically, we recommend that they--

- insure low teacher-pupil ratio essential for adequate individual attention;
- provide materials appropriate to the age and ability levels of students;
- implement a testing program that will benefit all segments of the school community;
- facilitate the development of a supportive relationship between community and school;
- provide assignments for beginning teachers that ease their introduction to the problems of teaching in urban schools.

Second, the professional establishment owes every teacher high-quality training. For urban teachers this must include training that is directed toward the particular problems of their classrooms and their charges. We call for preservice and inservice training that provides the following:

- methods for teaching, in a single classroom, students representing a wide range of ability;
- methods for implementing an activity-oriented program for students that will help them relate mathematics to their lives;
- methods for reaching the unmotivated student, for enhancing students' self-image, and for raising the academic aspirations and expectations of students;
- classroom management techniques that provide a setting for learning and that establish order and security for students;
- information about the special characteristics of urban students that will enable teachers to interact with them appropriately--for example, information about student language and the social dynamics of adolescents and their families in the specific community where the teacher is to work;
- experience in the social milieu of the community;
- special language instruction when appropriate;
- means to work within the system to the advantage of students;
- reasonable self-evaluation techniques that will help the teacher maintain a positive attitude in the face of such problems as administrative inertia and poor community attitudes.

NCTM owes the urban teacher more than generalized support and understanding. Specifically, we pledge to focus the Council's attention on urban education in the following ways:

- All Council committees will be directed to address the problems of urban mathematics education.
- The Council will form and fully support a task force on urban education.
- The attention of Council members will be directed to these problems at all NCTM meetings.
- Information on constructive solutions and programs will be communicated to Council members through NCTM journals and other publications.

We recognize that these actions still fall short of what needs to be done to solve all the problems of urban education. We are prepared, therefore, to take further action as additional options are identified.

(Reprinted by permission of the publisher.)

INTRODUCTION

The urban child and mathematics instruction: Is there a magic key to make the learning process more efficient for him or her? Are there organizational patterns, curricula, teaching strategies, materials, or some other factors which hold particular promise for increasing the urban child's mathematical achievement?

Panaceas are endlessly promised . . . but even money has not bought them and made them fulfill the promise. One reason is obvious: The urban child is a construct, a stereotype. Because we can identify where the child lives, we label the child. We obscure the variances of neighborhood, socioeconomic level, parental education, parental concern, of school buildings and administration, and teachers, of personality, self--that which helps one decide in favor of striving for schooling or fighting it. We cannot control most of these variances--but neither can we ignore the fact that they exist. The variances should make us focus on the individual--and the group of individuals that come into our schools. The group is individuals, and the decisions we make must be made with that in mind.

It seems evident to me, after years of reading research reports, that there are no significant ways in which urban children learn mathematics differently from non-urban children. There are a few teaching procedures that might facilitate learning for urban children but most procedures that are effective for urban children are effective for children in general. Many others are only specific versions of more general procedures. For instance, "use problem situations that mean something to the child" obviously might imply "city setting" to the urban teacher, just as it might imply "farm setting" to the rural teacher.

But the learning of mathematics proceeds for urban children as it does for other children. They go through the same stages of learning, and at relatively the same rate, give or take the same variance. They need most of the same type of content, depending on such factors as their abilities and goals--but the so-called "basic" skills or "survival" skills or "minimal competencies" probably differ little for children in different settings.

There are differences--in the mean attainment or achievement level. But only for some urban children--not all. The data rather clearly indicate that more than "urban" is the key--the type of "urban" setting must be clarified. And the data indicate that many forces impinge on the learning environment, not all of which can be controlled by the school. The statement by the Board of Directors of the National Council of Teachers of Mathematics which is included as a preface to this paper acknowledges such forces--and recommends some actions that could aid the teachers of urban children and help children as they teach and learn mathematics.

In the pages that follow, we shall look at some of the factors that have been considered in relation to the teaching of mathematics--organizing for instruction, curricula, what goes on in classrooms, evaluation, student characteristics, instructional materials. But first, the status of mathematical achievement of students in urban schools will be considered.

I. THE STATUS OF ACHIEVEMENT OF STUDENTS IN URBAN SCHOOLS

Data from the first assessment of mathematics conducted in 1972-1973 by the National Assessment of Education Progress (NAEP)* provide some comparative

* Four reports on the first testing have been published (NAEP, 1975a, 1975b, 1975c, 1976), in addition to a series of interpretive articles (Carpenter et al., 1975-76), a general statement of objectives (Norris & Bowes, 1970) yearbooks, and newsletters. About 90 thousand students at ages 9, 13, and 17, plus 4000 young adults ages 26-35, were tested. In addition to age levels, data were also analyzed in terms of sex, race, region, level of parental education, and community size and type.

evidence on the level of attainment of students who live in seven types of communities. The NAEP categories, which are based both on size of community and on type of employment of the majority of people in the community, are:

High metropolitan. Areas in cities with a population greater than 150,000 where a high proportion of the residents are in professional or managerial positions.

Low metropolitan. Areas in cities with a population greater than 150,000 where a high proportion of the residents are on welfare or are not regularly employed.

Extreme rural. Communities with a population under 3,500 where most of the residents are farmers or farm workers.

Urban fringe. Communities within the metropolitan area of a city with a population greater than 200,000, outside the city limits and not in the high or low metro groups.

Main big city. Communities within the city limits of a city with a population over 200,000 and not included in the high or low metro groups.

Medium city. Cities with populations between 25,000 and 200,000.

Small places. Communities with a population of less than 25,000.

Median differences from national performance for each community type at the four age levels tested are depicted in Figures 1 and 2.

Figure 1 indicates the median differences for "mathematics fundamentals" --computation and word problems with whole numbers, fractions, and integers.

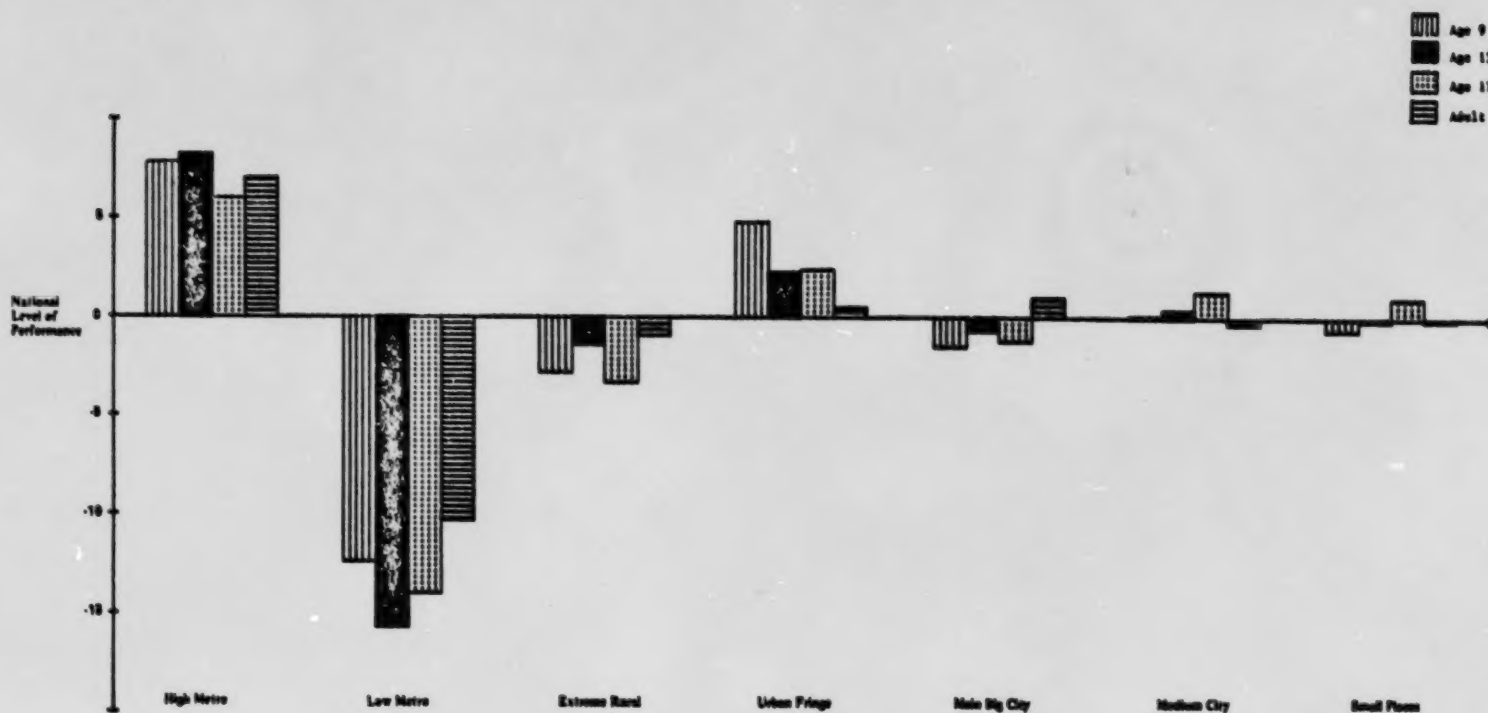


Figure 1

National Assessment of Educational Progress: Mathematics Fundamentals (1972-1973):
Median Differences from National Performance Levels by Size and Type of Community

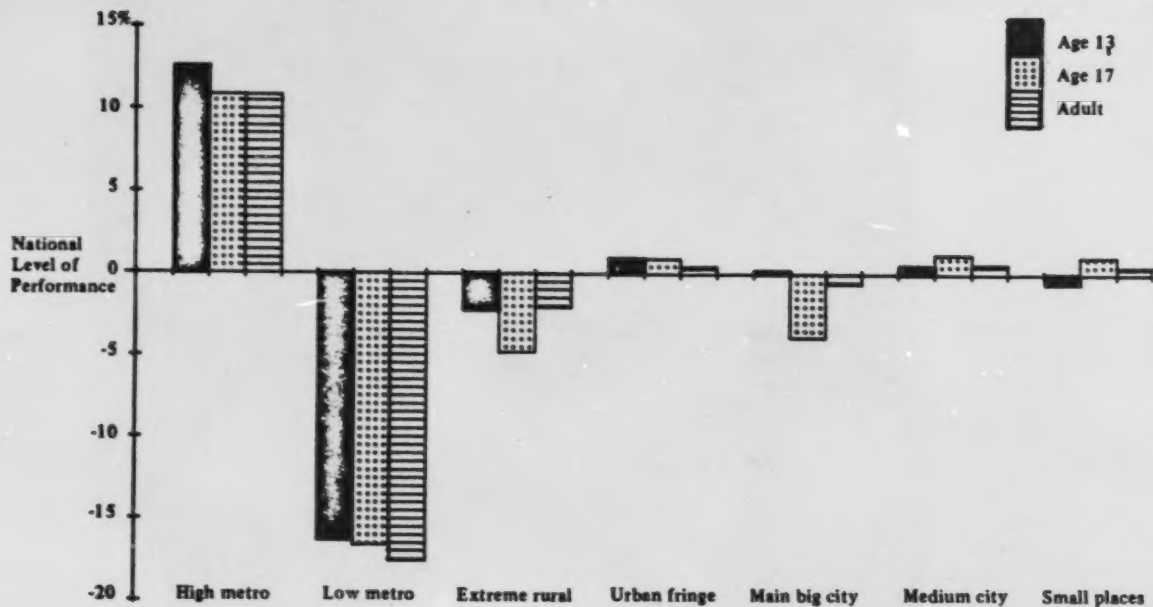


Figure 2

National Assessment of Educational Progress: Consumer Math (1972-1973):
Median Differences from National Performance by Size and Type of Community

Results for high metropolitan and low metropolitan communities

differed appreciably from national percentages. . . . The high metro group performed consistently above the nation on almost all exercises at every age level with overall results approximately 6 to 8 percentage points above national levels. Overall results for the low metro group were 10 to 16 percentage points below the national levels, showing the greatest difference from the nation at age 13. . . .

Low metro individuals generally showed the least difference from the nation on the simple computation problems on which national percentages were high. Although patterns were not consistent on every problem, 17-year-olds and adults were usually closer to the nation on simple computation exercises than the younger age levels. In contrast to their overall performance, 13-year-olds held a slight advantage over the older groups (when compared to national performance) on several addition problems.

Two other types of communities, the extreme rural and the urban fringe, showed differences from the nation although not as substantial as for the above two community types. At ages 9, 13 and 17, . . . the urban fringe performance was 2 to 5 percentage points above that of the nation. Results for adults in these two groups were quite close to national results.

Performances of main big city, medium city and small places respondents were very similar to that of the nation for all ages. (NAEP, 1975a, pp. 38, 40)

Figure 2 pictures the data on consumer problems. Again the variance was greatest for high and low metropolitan students.

. . . The median differences for the high-metro group were from 11 to 13 percentage points above the nation; for the low-metro group, from 16 to 18 percentage points below the nation. The high-metro group held the greatest advantage at age 13, while performance of the low-metro group was furthest below that of the nation at the adult level.

. . . for the main-big-city group; 13-year-olds and adults were very close to national performance while 17-year-old results showed a greater deficit.

Results for the urban-fringe, medium-city and small-places groups were very close to that of the nation, with median differences generally less than 1%. (NAEP, 1975b, p.34)

From the data on both types of items, it is apparent that performance differed not only in terms of community size, but in terms of socioeconomic factors (which pertained to the students' parents and other adults in the community). This confirms the pattern that was anticipated by many agencies, educators, and others: The mean achievement of students who have in the past been identified by labels such as "inner-city" or "disadvantaged" is below the level attained by most other sets of students.

In a recent analysis, Bright (1978) pointed out that assessment data can be viewed from another angle. While he could not analyze the data in terms of subpopulations, he did study total-group data from NAEP and several other assessments to determine patterns. He reported:

Overall several patterns in the data seem to support clear conclusions. First, there is general improvement in performance across grades. This result is not unexpected, and it is consistent with the results of the grade-equivalent studies discussed earlier. Second, the levels of performance decrease as the items become more complex. Third, performance tends to stabilize. For the areas discussed in this article, stabilization seems to occur during the junior high school years Fourth, stabilization of performance for whole number computation occurs earlier and at a higher level than for fractional number computation. . . . Fifth, for all computation skills considered, there is no decline--or at least no important decline--in the performance of adults when compared to that of high school students. In the context of improvement of skill performance across grades, this suggests that once skills are mastered they are not forgotten.

. . . (it is observed) that computation skills are not acquired on the basis of initial instruction. Instruction over several years is needed to reach stability, and in every area examined there is still room for improvement

It is important to note that the data presented refute the notion that students generally do not acquire basic computation skills. In fact, some skills (e.g., addition and subtraction without regrouping) are almost universally acquired, while other skills (e.g., division of decimal fractions) are not. Any meaningful discussion of the performance of students in basic computation skills must be a discussion of specific skills rather than skills in general. (p. 160)

While all of Bright's conclusions are not directly drawn from the data on the "low metropolitan" students assessed by NAEP, most of his comments nevertheless seem applicable to such students. Bright's analysis also indicates that the evidence on skill attainment must be studied carefully, not only for the causes and implications of declining scores, but also for patterns of learning.

The most widely cited instance of declining scores concerns the Scholastic Aptitude Test. Decreases in scores were observed; the average score of mathematical ability was 502 in 1963 and only 472 in 1975. While the test scores were for college-aspiring students, much of the mathematics tested is of a basic nature. Therefore, declines in scores are presumed to be symptomatic of a failure to establish competency in mathematics, although it has been pointed out that the "Ultimate blame may rest with the influence of television, permissive parents or dozens of factors beyond the control of schools" ("Those Dropping Test Scores," 1975, p.34). It has been noted that scores on the language (verbal ability) portion of the SAT were even more depressed than those for mathematics.

Harnischfeger and Wiley (1975) analyzed scores from nine widely used testing programs, including both elementary-school and secondary-school tests: the SAT, the American College Testing Program (ACT), the Iowa Tests of Basic Skills (ITBS), and six others. Nearly all reported data showed declines for Grades 5 to 12 over the past decade. Both the verbal and mathematics scores on the SAT peaked in 1963 and then declined steadily. On the ACT a similar pattern was found, and on the ITBS, the pattern was one of general increase from 1955 to 1963, then consistent decline to 1970.

They traced likely causes for the drop in achievement levels to both home and school, but indicated their belief that school-related factors--such as the number of students taking mathematics courses, the increasing absentee rate, and the larger percentage of students taking tests who once would have dropped out of school--can be more closely studied and more easily influenced.

It can be presumed that the NAEP "low metropolitan" group, for whom attainment was below the median, contained a high percentage of blacks. The data, when analyzed by race, indicate that blacks performed 14 to 21 percentage points below the national average, whites performed from 3 to 4 points above the national average. The difference was smallest at age 9 and increased for 13- and 17-year-olds with no appreciable change between ages 13 and 17. On consumer-math problems blacks were 20 percentage points below at age 13, 24 points below at age 17, and 29 points below as adults; whites were 4 to 5 points above the national level.

In another type of national survey, Okada et al. (1969) cited data from the Educational Opportunity Survey on the achievement of black and of white students. Blacks did not attain the sixth-grade achievement level for mathematics until Grade 8. From Grades 6 through 12, there was a gradually increasing gap between black and white students, with similar lags in achievement observed for other "disadvantaged" groups.

Such results are also evident in many smaller-scale studies. They reflect a pattern of attainment, but at a different, later point in time. They raise the possibility that the expectation is unjustified that all students should attain competency at the same time. They also lead to conjectures about the validity of what is being tested--and the way it is being taught.

II. EVIDENCE ON EXISTING PRACTICES IN MATHEMATICS INSTRUCTION

Comparatively little research in mathematics education has focused directly on comparisons of urban and non-urban students. On the other hand, many research findings pertain to students in general--samples from all types of communities were included in the studies and, collectively, they have implications for urban as well as non-urban schools.

In a recently completed review of the literature on existing practices in mathematics education between 1955 and 1975, Suydam and Osborne (1977) analyzed published evidence related to both curriculum and instruction. A significant proportion of the information in this section is drawn from that report, as well as from previous reviews by Suydam and others.

Organizational Patterns

Educators have long searched for one "perfect" way of organizing schools and classrooms to meet the needs of individual students and to increase achievement. Much has been written about such organizational patterns as departmentalization, the use of mathematics specialists, multi-graded and non-graded approaches, team-teaching, "open-space schools" and "open-classroom environments," alternative schools, and diverse modular scheduling patterns. The 1977 NCTM Yearbook (Crosswhite, 1977), for instance, was focused on organizing for mathematics instruction. While many organizational patterns are in use, perusal of a wide variety of documents results in the conclusions that the graded, self-contained classroom at the elementary-school level and the fixed-period schedule of the secondary school remain the predominant patterns.

Over the years, a large number of studies has been conducted to ascertain the efficacy or the superiority of one or another pattern. Such attempts have been hampered by the difficulty of isolating and measuring the effects of the organizational pattern, since such factors as curriculum and teacher skill interact with the pattern. Additional confusion results because definitions of the various patterns tend to overlap, so that, for instance, what one person labels team-teaching, another may define as departmentalization.

Reviews of research (e.g., Suydam, 1972; Suydam & Weaver, 1970, 1975), lead to the conclusion that there appears to be no one organizational pattern which will increase student achievement in mathematics. Proponents of any pattern can find studies which verify their stand, while a large proportion of the studies reported no significant differences in achievement between two or more patterns. Decisions appear to be made on the basis of selected evidence and a hope for improvement. It appears that belief in a particular pattern and a desire to make it succeed may aid in creating an environment conducive to increasing student achievement and teacher satisfaction. The specific components that make any organizational pattern effective and the weaknesses that cause another pattern to seem less effective are rarely documented; rationales rather than evidence abound. Perhaps the most important implication of the various studies is that good teachers can be effective regardless of the nature of the school organizational pattern. Concomitant with this is the strong suspicion that some teachers can be more effective with one pattern than with another.

There is also an implication that efforts to change school and classroom organizational patterns should not be attempted with the expectation of unqualified success in effecting long-range change. Nor should change or organizational patterns be promoted as an assured means of increasing mathematical achievement. Rather, such efforts should be acknowledged as a vehicle for helping teachers work toward improvement of instruction.

Curricula

In the 1950s and 1960s, many curricula and programs were developed by staffs of specialists. Materials such as those developed by the School Mathematics Study Group (SMSG), the Madison Project, the Greater Cleveland Mathematics Project (GCMP), the Stanford Mathematics Project, Minnemath, the University of Illinois Committee on School Mathematics (UICSM), and the Individually Prescribed Instruction Project (IPI), among others, have been widely reported

(e.g., see Suydam & Riedesel, 1969). The content of mathematics was extended, resequenced, and presented in new ways in these programs.

In the later 1960s, the Secondary School Mathematics Curriculum Improvement Study (SSMCIS), the Comprehensive School Mathematics Program (CSMP), and the Unified Science and Mathematics for Elementary Schools Project (USMES) were funded; materials from all are available in ERIC. Myriad other projects have been funded, including the following ones, largely developed in the 1970s. They are cited in Lachat et al. (1977) as "programs that work."

Academic Advancement Program (AAP) (Morristown, New Jersey)

A program of individualized, performance-based, self-paced instruction in mathematics, employing the principle of mastery learning, for grades 6-8 and remedial use in grades 9-12. The program is organized into 66 hierarchical units, integrating four commercially produced mathematics series to give a variety of options. Its purpose is to increase the learning rate of students in a heterogeneous classroom, meeting individual needs within a socially integrated context. Experimental students generally made higher gains than control students in homogeneous groups.

Bilingual Continuous Progress Mathematics (SEDL) (Austin, Texas)

A bilingual developmental program which provides basic mathematics instruction in an aural/visual mode, for first-grade, Spanish-speaking children who do not read or understand English (especially migrant children). It is based on 93 objectives arranged into mathematical areas at two levels of difficulty. It provides for the mastery of sequentially arranged mathematics objectives; allows student participation, independent of reading ability; provides for individualized instruction and progress; allows migrant pupils to participate fully during their shortened short term; and introduces mathematical concepts through concrete manipulative experiences. The project group showed statistically significant gains and made significantly greater gains on the test than the control group. Students in the program had little difficulty moving into traditional second-grade mathematics programs.

Conceptually Oriented Mathematics Program (Columbia Public Schools, Missouri)

A diagnostic/prescriptive continuous-progress mathematics program for grades 1-8. It is designed to help students master skills in a sequence allowing for a wide range in competency and achievement, and to provide a framework for diagnosing children's needs and prescribing appropriate activities. It is developed with the beliefs that (1) teachers will break away from a lock-step textbook sequence if given tools for diagnosis and prescription, and (2) commercially produced textbooks can be used effectively in individualized programs. Evaluation in grades 2-8 showed that the mean increase after one year in the program was 10.5 percentile points, with 77% increasing 16.5 points and 20% decreasing 10.5 points. No significant differences were found in two other districts.

Developing Mathematical Processes (Rand McNally & Company)

An activity-oriented continuous-progress elementary mathematics program, the mathematics component of the Individually Guided Education (IGE) program of the Wisconsin Research and Development Center. Ninety topics are built around a set of hierarchically-related objectives, clustered into content areas. Progress need not be in a linear order, allowing for a variety of learning routes and instructional groupings. It is designed to give children in grades K-6 the opportunity to investigate actively mathematical aspects of the world around them, to give them the opportunity to move slowly from the concrete to the abstract in skill development, to broaden the scope of problems they can solve, and to capture children's interest through a variety of physical materials and activities. It is based on the premise that active participation in learning, rather than passive reception of knowledge, helps develop a child's ability to think. Pretest-posttest gains of project students were significant at the .05 level. No significant difference was found between the scores of students who used the DMP materials and the scores of similar students using another program, but the direction of the differences generally favored the DMP groups.

Distar Arithmetic (Distar Instructional System, Science Research Associates)

A basic arithmetic program in three consecutive levels designed to accommodate individual pacing of skill development, for preschool through grade 3 but appropriate

for any preschool or elementary grade level lacking basic arithmetic skills. Its purposes are to establish specific criteria of presentation, to set the same educational objectives for all children, regardless of learning rate or ability; to provide an ungraded curriculum and individualized instruction, to provide opportunities for acceleration for fast learners and recycling procedures for children needing skill reinforcement. Data and additional discussion of this program are given later in this review.

Educational Developmental Laboratories Mathematics System (EDL)
(McGraw-Hill)

An ungraded, continuous-progress mathematics system for grades 1-6; it is also designed to be suitable for adults and secondary students needing remediation or a developmental program. Organized into a series of 156 instructional cycles, it provides self-paced, diagnostic/prescriptive learning through multi-media devices to motivate students. In one study in a junior high school, the experimental group's gains were more than twice the control group's seven months of use.

High Intensity Tutoring (HIT) (Highland Park, Michigan)

A remedial basic skills program using cross-age tutoring with a motivation component. Tutees are students in grades 6 and 7 performing one to five years below grade level, while tutors are in grades 7 and 8. It uses highly structured materials for students with special reading and mathematics needs. It requires highly qualified and respected teachers, and uses rewards as incentives. Participants made significant gains on the Wide Range Achievement Test.

Project R-3 (San Jose, California)

An interdisciplinary junior high school program emphasizing highly motivational instructional approaches. It is designed to upgrade performance through motivating techniques and materials, to raise student occupational and aspirational levels, to improve overall classroom and school social behavior, to provide measures for the students' parents and families to participate in the program, to incorporate students' cultural strengths into the school program, and to enable school staff to acquire an understanding of the special characteristics of R-3 students. Gains made by the R-3 students were significantly greater than those of control groups, and greater than growth rates reflected by norms.

The overall mathematics achievement gain was 2 years for eight months in the program. Highest gains were in Computation and Concepts (2.3 and 1.9); the lowest gain was Applications (1.3).

Project Sail: Student Achievement in Individualized Learning
(Wharton, New Jersey)

A diagnostic/prescriptive individualized mathematics program with topics ranging from whole number place value to pre-algebra concepts for grades 6-7. It is designed to improve mathematics achievement for students at all ability and achievement levels, to provide an eclectic approach to learning, and to provide for self-pacing. It seeks to create a stimulating environment suited to individual needs for different learning techniques and materials, emphasizing encouragement, praise, and faith in students' abilities. Results from a three-year evaluation period were consistently positive; students exposed to two years of Project SAIL significantly outperformed nonproject students in total mathematics achievement, and significantly improved their ability to perform computations.

Project Smart: Success in Mathematics Through Aural Reading Techniques (Volusia County Educational Development Center, Daytona, Florida)

A completely individualized mathematics program in fundamental computational skills, with major emphasis on aural presentation of text materials via audio-cassette tapes. Level I is for fifth and sixth-grade students deficient in computational skills, while Level II covers basic mathematics curriculum for fifth through ninth grades. It is designed to enable students to develop computational skills commensurate with mathematical aptitude rather than reading ability, allowing the teacher time to work with students individually by providing each student with an alternative source of aural information. Difference in gains on tests between experimental and control groups significantly favored project students.

Systems Approach to Individualized Instruction (Manzanita Elementary School, Grants Pass, Oregon)

An individualized, ungraded basic skills curriculum in reading, language, and mathematics based on performance objectives, using a differentiated staffing pattern for students in grades 1-6. It provides a curriculum system applicable to diagnosed student needs, emphasizes systematic and preventive instruction, and provides for vertical

articulation based on individual student needs, interests, and learning styles. Data indicated a statistically significant gain at each grade level.

Diagnostic Prescriptive Arithmetic (Staten Island, New York)

A diagnostic-prescriptive activity-oriented approach to basic arithmetic concepts and skills for students in grades 2-6. It is designed to increase a student's rate of learning in arithmetic and to meet specific needs through prescriptive activities oriented to individual and small group instruction. It emphasizes the use of materials and activities which encourage the learning of mathematics through creative experiences. Results indicated that the mean posttest level was raised above the expected mean, with averaged increases of 1.5 grade equivalent units in the seven months from pre- to posttest.

In addition to complete curricula, many projects developed supplementary programs.

Individualized Computational Skills Program (Houghton Mifflin Company)

A supplementary ungraded mathematics program for use with any basic mathematics program in either a graded or ungraded setting, for students in grades 1-2.

Infinity Factory (Education Development Center, Newton, Massachusetts)

A children's television series about mathematics for children between 8 and 11 years old, to help children understand the usefulness of mathematics in their own lives, to present content through a positive ethnic perspective that will help children share and appreciate different cultures, and to help children from ethnic minorities become more comfortable with mathematics.

Multi-Resource Mathematics Management System (MRMMS) (Linden Avenue School, Glen Ridge, New Jersey)

An intensive diagnostic prescriptive program of mathematics instruction for children in K-4.

Pre-Algebra Development Center (Chicago, Illinois)

An eight-week pre-algebra mathematics program using a four-phase approach to individualized instruction, for eighth-grade graduates deficient in skills necessary for ninth-grade algebra.

Project Catch-Up (Newport-Mesa Unified School District, Newport Beach, California)

A program of individualized instruction based on criterion-referenced objectives and conducted in a laboratory setting for underachievers in grades K-6.

School Volunteer Development Project (Do It) (Dade County Public Schools, Coral Gables, Florida)

A volunteer tutoring program based on a systematized approach to using volunteer services for students in grades 2-6.

Strategies in Early Childhood Education (SECE) (Waupun, Wisconsin)

An individualized developmental program designed to build essential reading, language, and mathematics skills, in grades K-2.

Teams-Games-Tournaments (Baltimore, Maryland)

An instructional method for basic skills that reorganized the classroom into heterogeneous ability grouping, for students in grades 3-5 (in mathematics). The class is divided into four or five-member teams; skill game exercises are played during tournaments.

These and other curricula, including those exemplified by other commercial textbooks, provide alternatives. The mathematical content, however, is not at issue in the majority of these programs; that is, the scope of content presented differs relatively little from program to program. Sequencing and manner or mode of presentation--that is, methodology--are the primary variables. One program does differ substantially in the mathematics content it includes--and this in turn necessitates other changes. The Comprehensive

School Mathematics Program (CSMP) focuses on mathematical ideas rather than computational skills per se at the elementary school level, and emphasizes the structure of mathematics in its secondary school materials. Evaluation has been continuous, with full reports found in ERIC to support achievement claims.

Class Size

Educators believe that class size is an important variable affecting the quality of teaching and learning and thus affecting achievement. But there is little evidence that mathematics achievement is affected in a simple or direct way by total class size; rather, the size of the group with whom the teacher works on a particular topic may be of importance. A ratio of one teacher to one pupil (e.g., Moody et al., 1973), while seeming optimal by some criteria, obviously does not seem optimal by other criteria, not the least of which are fiscal limitations. To a greater extent today than in previous decades, class size is negotiable by teachers with school boards. But as school budgets tighten, the number of pupils per teacher, which had decreased by the end of the 1960s, has begun to climb upward again. In some sets of data, however, this is obscured by including special class, supplementary services, administrative, and other personnel in the equation.

Furno and Collins (1967) analyzed data from 16,449 pupils enrolled in third-grade classes in Baltimore schools in 1959; and traced their patterns for 1959-1964. Students in smaller classes in the regular curriculum made significantly greater gains in arithmetic achievement over the 5-year period in 96 comparisons to 29 for students in larger classes. The advantage of small size (up to 25 students) was considerably greater for non-white students and for those in the special education curriculum.

In discussing data from the National Longitudinal Study of Mathematical Abilities (NLSMA), Begle (1973) reported that class size (less than 30,

or greater than or equal to 30) had an effect on achievement in 8 or 16 instances. He commented: "Curiously enough, the smaller class size was more advantageous for elementary school students, but the larger class size was more advantageous at the junior high school level" (p. 212). Salopek (1974) reported that class size was one of 3 consistent predictors of variance on arithmetic tests in Grade 6 in one county in Pennsylvania, and similar correlational data have been reported from a few state assessments of achievement.

Jamison et al. (1974) concluded, however, that "where significant differences were found they were about as likely to favor large classes as small and that even when differences were significant they were usually small" (p. 21).

Time Allotment and Use

The amount of time spent on mathematics instruction and how that time has been allocated is a matter of increasing concern. In studies in which teachers were asked to indicate the amount of time spent on mathematics instruction, data on time allotments suggested by state guidelines were affirmed (Graeber et al., 1977; Miller, 1958; Jarvis, 1966; Price et al., 1975), although variance was great within grade levels. Thus in Graeber et al. (1977) the range was 15 to 25 minutes to over 65 minutes in both Grades 1 and 3, and from 26 to 35 minutes to over 65 minutes in Grades 5 and 7. It was also noted that the lower the grade level, the less time was spent on mathematics. Several recent studies (Conant, 1973; Filby et al., 1976; Fisher et al., 1976a, b; Marliave et al., 1976) cited data from classroom observations. They indicated that far less time may be spent on actual instruction than is reported. From other studies (e.g., Olson, 1971), it appears that approximately 20% of the elementary-school day is allocated to mathematics instruction; at the secondary-school level, 200 to 300 minutes per week.

How time is used may be of more importance than how much time is available. To determine how the use of class time affects achievement, Shipp (1958) compared four groups, in which 75%, 60%, 40%, or 25% of class time was spent on developmental work while the remainder was spent on individual practice. Higher achievement in computation, problem-solving, and concepts was obtained when more than half the time was spent on developmental activities. In replications of this experiment, Pigge (1964) and Zahn (1966) used other time allocations at different grade levels. They confirmed that when the greater proportion of time was spent on developmental activities, achievement was higher. Graeber et al. (1977) provided data on how teachers say they spend time.

In grades three, five, and seven it is interesting to note that approximately 30% of the teachers report that they spend 10 minutes or less introducing new work. Also, very little class time reportedly was used for review of concepts and drill in basic facts [or algorithms]. Twenty-eight percent of the fifth-grade teachers and 48% of the seventh-grade teachers reported that they use less than 6 minutes of daily class time for review or drill. (p. 13)

It comes as a surprise to many people that there are actually relatively few studies which describe the actual classroom situation. Goodlad (1977) noted:

There is only one honest answer to the question, "What goes on in our schools?" It is that our knowledge is exceedingly limited. . . . There is not now either a body of data on what transpires in schools from which to begin an enlightened discussion of schooling or a tested methodology for securing these data. (p. 3)

In most studies in the classroom, the setting is described only generally. Comparisons are made with the "traditional" or "usual" classroom, as if everyone knew precisely what that was. There are also some surveys in which teachers were asked to list or to check activities which they use. But only rarely have observers gone into classrooms to see and define what is

occurring. Some studies provide information on verbal behaviors (e.g., Fey, 1969; Halperin, 1976; Kester, 1969; Mahan, 1971; Meckes, 1972; Stilwell, 1968). Thus, in the classrooms studied it is known that:

- The teacher talks about 2/3 of the time.
- Teachers tend to use a direct, rather structured approach.
- Over 50% of the questions teachers ask are at the knowledge level, requiring relatively low-level cognitive processes from students.
- The teacher initiates most exchanges, with students doing little more than answering questions--in addition to sitting and listening.
- Teachers communicate with brighter pupils in a more friendly and encouraging manner than with other students.

Evidence from a variety of sources documents a picture of the classroom--at both elementary-school and secondary-school levels--that has changed little, despite the innovations advocated in the past 20 years (e.g., see Alspaugh, 1966; Brown, 1974; Conant, 1973; Gates, 1969; Goodlad et al., 1970; Hughes, 1959; Price et al., 1975; Shetler, 1959):

- Teachers spend a large proportion of their time on non-instructional duties; an "astonishing amount of time" (perhaps up to 70%) is taken up in control, managerial routines, and what appeared to be scarcely more than busywork.
- Telling and questioning, usually in total-class groups, is the prevailing teaching method.
- Tell-and-show and seatwork at the elementary-school level, and homework-lecture-new homework at the secondary-school level, are the prevailing patterns.
- Textbooks predominate as the medium of instruction, with a single text followed closely; some teachers use virtually no other activities or materials.
- The pace of lessons is slow, yet teachers allow little time for individual pupils to answer questions.

- In the elementary school, the major portion of time is spent on reading and language arts, with mathematics second.
- Seatwork consumes up to 50% of the time in class; questions and answers, or discussion and explaining involve about 25% of the time.
- Teachers are teaching essentially the way they were taught in school.

In an interesting variation on the usual assessment of time allocations, Barley (1975) described the amount of time in formal instruction in terms of the percentage of their waking lives that students spend on each subject. For the majority of students K through 12, school days consume 16% to 20% of their waking lives in any given school year. The majority of elementary-school students spent about 3% of their waking lives on mathematics (compared with 7.6% on reading and language arts). Secondary-school students spent about 1.8% of their time on mathematics (compared with 2.1% for science).

Harnischfeger and Wiley (1975) concluded that students who spend 190 days in school achieve more than do those who spend only 170 days in school each year. In commenting on this, Goodlad (1977) stated:

If the time spent on learning affects quality and quantity of outcomes . . . then how the days and weeks of the school years are being used and how they might be used differently become first and second items on the agenda of school improvement.
(p. 4)

Earlier, he had noted:

To carry on a serious dialogue about, let alone to seek change, American schooling or simply the local elementary school, without a rather substantial body of the information implied seems somewhat bizarre. And yet, to do so is virtually a national pastime. In our pseudo-wisdom, we know what schools need without knowing what they already have and we know what to put into them without knowing what needs to be replaced. (p. 3)

Achievement differences favored students in Grades 4 to 6 spending 60 minutes per day rather than 40 minutes per day on mathematics instruction (Jarvis, 1963; Lawson, 1966). In the Oregon (1976) progress assessment, however, amount of time per day in formal mathematics instruction (16-30 minutes, 31-45 minutes, 46-60 minutes) resulted in little or no significant difference in performance. The percentages allocating each amount were 12%, 50%, and 32%.

Doubling the length of the class period from 55 minutes to 110 minutes by meeting for only half the number of periods was not found to affect the achievement of secondary-school students (Albers, 1973; Hansen, 1963). Whether lengthening the number of semesters spent on a course has an effect on achievement has been studied with algebra for low achievers; results differed in studies by Buchman (1972), Herriot (1968), and Posamentier (1973).

Teaching Approaches

Many varying instructional approaches have been and are being tried in classrooms. The literature reflects current concern that far too many of them have been promoted as panaceas, rather than as components in a teacher's repertoire, to be used as children, content, and circumstances warrant. The emphasis of research has been on such comparisons as expository versus discovery approaches or incidental versus systematic procedures. Only limited attention has been focused on the circumstances under which each could be optimally used by an individual teacher. If children learn best in various ways, then individual teachers may teach best in various ways and specific content may be taught best in certain ways.

The topic of most concern for the past two decades, discovery learning, has been the object of several reviews. Tanner (1969) found studies on discovery versus expository instruction provided "insufficient rationale

for sweeping changes in curriculum and instruction" (p. 655). In his more extensive study, Weimer (1975) also reported no clear evidence of a single superior method; rather, he said that many effective teaching strategies are available.

Suydam and Weaver (1970, 1975) concurred with this; a summary by Robertson (1971) expresses their point:

It would appear that no one treatment or mode of instruction can be considered the best approach. The teacher who learns as many instructional modes as possible, identifies and diagnoses pupil needs and abilities, and uses this knowledge to individualize instruction may very well get the best results. (p. 5279)

Research has indicated rather clearly, however, that meaningful instruction (that is, instruction in which the learner is taught understanding of an idea) will lead to higher achievement than will rote instruction (Weaver & Suydam, 1972). This does not preclude all learning by rote, however, for certain skills are particularly amenable to such procedures.

Learning through activity approaches, such as a mathematics laboratory, was stressed increasingly in the 1960s. There is evidence that teachers believe that activities should be used--yet few teachers, especially at upper levels, actually use them. Students using activity-oriented programs or units can be expected to achieve as well or better than students using programs not emphasizing activities (Kieren, 1969, 1971). That the mathematics laboratory is one strategy among many, to be used as appropriate, has been noted frequently.

Research has also been concerned with a larger number of specific comparisons of techniques to be used in teaching; for instance, subtraction with regrouping, the division algorithm, or algebraic equations. Such studies have been summarized in a variety of documents with interpretations

for classroom applications (e.g., Callahan & Glennon, 1975; Suydam & Dessart, 1976; Suydam & Weaver, 1975).

In an overview of research on the effectiveness of various modes of instruction, Jamison et al. (1974) analyzed studies on traditional instruction. They concluded that "few variables consistently make a difference in school performance" (p. 26). They emphasized that this "does not, however, imply that schools make no difference in the cognitive development of their students" (p. 27).

In a different type of study, Hartley (1978) used meta-analysis. She synthesized 153 studies comparing traditional instruction in mathematics with computer-assisted instruction, cross-age and peer tutoring, individual learning packets, or programmed instruction. On the basis of these studies, she concluded that tutoring was "the superior technique for increasing mathematics achievement" (p. 4003). Peer tutoring was as effective as paid adult aides, while cross-age tutoring was slightly better than the other two. CAI was less effective than tutoring, but considerably more effective than either individual learning packets or programmed instruction. These two techniques were essentially comparable to traditional instruction.

Differentiation of Instruction

A major emphasis for over a decade has involved concern for individualized instruction. As Schoen (1977) reported:

In the 1960 and 1970s there has been renewed emphasis on the responsibility of schools to meet the needs of individual students. Not since the peak of the progressive education movement have educators focused so directly on the individual. This phenomenon is reflected in the professional education literature of recent years. For example, in 1971 the Education Index listed 124 articles on individualized instruction; the average number was about 35 a year during the decade of the sixties. In contrast, only about 4 or 5 (and often fewer) articles on individualized instruction appeared each year in the forties and fifties. (p. 198)

Belief in the need to account for individual needs, combined with ideas from learning theory and from technology, led to the development of self-paced programs of mathematics instruction. Research evidence fails to indicate that such programs are generally any more effective than other programs. Schoen (1976a, 1976b, 1977) has documented this point rather extensively; Kozak (1975) and Miller (1976) have also prepared recent reviews. Schoen (1977, pp. 212-213) summarized his findings:

- Results favored traditional instruction (TI) more often than self-paced instruction (SPI), although many analyses resulted in no significant differences.
- Locally developed programs were about as effective as those sold commercially.
- SPI was particularly ineffective in developing computational skills at the intermediate and junior-high school levels.
- High-ability students achieved equally well in SPI as in TI, but most low-ability students were unable to function in SPI.
- On affective criteria, 40 of 55 analyses resulted in significant differences between the TI and SPI groups. SPI groups were consistently superior only at the primary-grade levels.
- Teachers in SPI have tended to spend more time on procedural matters and to restrict educational discussion more specifically to the topic at hand than teachers in a typical TI classroom.
- The diagnosis-prescription aspect of SPI has not been shown to be effective.
- Projected over a five-year period, costs for materials alone for SPI were about four or five times that of TI.

Two independent studies, one funded by NIE and conducted by the Educational Testing Service and one funded by the Office of Education and conducted by American Institutes for Research, also suggested that materials for

individualized instruction "made little or no difference in improving achievement unless the materials are used in a setting where there is one-to-one interaction between teacher and learner . . ." (EPIE, 1977, p. 2).

Many procedures for differentiating instruction have, however, been found to be effective; for instance, grouping for specific needs. While research evidence tends to be equivocal (Suydam & Weaver, 1975), there is evidence from individual users that it is a useful way to provide for individual needs. However, many teachers find it difficult to group for mathematics instruction; in the elementary school, grouping for reading has long been the pattern, but additional grouping for mathematics has not been widely practiced. In the secondary school, there is the long-held belief that one or another way of tracking students--that is, assigning them to classes by ability or achievement levels--will take care of the need for individualization. Thus, while many variations have been proposed, most elementary schools used heterogeneous grouping procedures, while secondary schools have used one or another form of homogeneous grouping.

The teacher must identify various factors related to pupils' achievement and interest in mathematics, and then decide on appropriate variations in content, materials, method, and time. Related to this is research evidence that, despite the fact that a teacher might be sensitive to and state differences among pupils, teachers frequently do not differentiate instruction, and may frequently select topics and ideas which students already know.

Strickmeier (1971) studied patterns of teacher verbal behaviors in seventh-grade mathematics classes grouped by ability; comparisons were made of teachers' perceptions of their verbal behaviors and expectations for classes of different ability levels. Although teachers had different perceptions and expectations for classes of different ability levels, such differences were not reflected by observable differences in the teachers'

verbal behaviors. Skager (1969) found that teachers selected instructional objectives for low-achieving seventh graders (in urban schools) that reflected skills already available to their students, and directed instruction to skills which were relatively well-developed when students entered the program, rather than at areas in which students were initially weak.

Stiglmeier (1973) similarly found little relationship between eighth-grade teachers' judgment of student needs and instructional mode. The Educational Products Information Exchange (EPIE, 1976b) reported a pilot study: "Students tested on the first day of school have achieved a mean score of 64 percent on tests made up of test items taken directly from the major materials from which they were to be instructed for the rest of the school year!" (p. 2).

Nelson (1960) interviewed 183 Nebraska secondary school teachers, visited 85 classes, and obtained written responses from 2,185 students. She concluded that the teachers used a wide variety of methods, but only the most capable teachers used techniques for adapting instruction to students' abilities and needs. Differences in teachers' skill rather than the method used were most evident. She observed "few accommodations to individual differences" in the organization of the classes.

Student Characteristics

Certain student characteristics seem particularly relevant as they interact with mathematics learning and instruction.

Student aptitude. Most of the research indicated that aptitude, as measured by intelligence tests, is highly correlated with mathematics achievement. This is hardly surprising, and one wonders why so much attention has been devoted to confirming the correlation between scores from intelligence tests

and scores from mathematics tests. What mathematical ability consists of has been the focus of a smaller body of the research.

Feierabend (1960) compared 19 studies concerned with the relationship of general intelligence and special abilities in mathematics. She concluded that:

Studies in this area appear to agree on the importance of a general intellectual factor for ability in mathematics, but the investigation of specific abilities is not conclusive and the approach to this problem is perhaps not as meaningful as it could be. . . . The question remains unanswered as to whether all persons of sufficient general intelligence have equal potential for mathematics, or whether there may not exist some special abilities, factors, or conceptual approaches which are specific to the field of mathematics or perhaps to creativity in mathematics thinking. (p. 36)

Aiken (1971), in an analysis of studies reported after Feierabend's review, supported her conjecture concerning the importance of special mathematical abilities, in addition to general intelligence, for achievement in mathematics. He also suggested that such factors as language, sex, age, and heredity need further study. He found that "only about half the variance in mathematical achievement can be accounted for by differences in abilities."

Among other conclusions Aiken stated were:

- There is some support for a broad group factor of mathematical ability, but generally it appears that mathematical ability, rather than being a unitary trait, consists of a number of facts.
- Individual differences in mathematical ability increase at successive age or grade levels (the range may be as great as seven years).
- Intra-individual changes in mathematical ability as a function of age have been extensively explored by Piaget.
- Such factors as prior experience and verbal ability have been related to mathematical ability, in addition to reasoning ability and spatial ability.

- Most studies on aptitude-treatment interaction have not indicated that, for an individual having a particular pattern of abilities, certain techniques of instruction are more effective than others.

As part of a study to determine ways to increase the learning of disadvantaged children, Stodolsky and Lesser (1967) reported on patterns of four mental abilities (Verbal Ability, Reasoning, Number Facility, and Space Conceptualization) in six- and seven-year-old children from differing social-class and ethnic backgrounds--Chinese, Jewish, Negro, and Puerto Rican. They found that ethnicity was reflected in the patterns of abilities:

Ethnicity does affect the pattern of mental abilities and, once the pattern specific to the ethnic group emerges, social-class variations within the ethnic group do not alter this basic organization. (p. 567)

They concluded that instruction in mathematics should teach to the strengths of each ethnic group--"even at the expense of magnifying the differences among the groups" (p. 588). In a similar vein, Green (1978) pointed out that, for traditionally raised American Indian children, a traditional, "non-participatory" methodology is better than one emphasizing discovery strategies.

The performance patterns of Mexican-American seventh-grade students on five mathematics tests were compared with scores of a predominantly Anglo group of students by Begle (1972). The Mexican-American group scored slightly below the Anglo group on the first three tests. None of the correlations between test scores was significantly different from zero for the Mexican-American group, while all but one were positive for the Anglo group. Begle commented:

This is not the first time that an SMSG study has suggested that the intellectual functioning of Mexican-American children is different from that of Anglo children. . . . it is not wise to generalize from what we know about the way Anglo children learn to children of other cultures. (p. 5)

In several locations, bilingual mathematics programs have been developed (see, for instance, page 15). Several studies (e.g., Begle et al., 1975) have reported that Hispanic students in bilingual classrooms achieved better than Hispanic students in regular classrooms and the reactions of those having bilingual instruction were highly favorable. The differences in the programs might have involved language or affective factors alone, however; the available information does not indicate whether the curriculum or the instruction was actually changed to take advantage of ethnic strengths.

Attitudes. Many people believe that mathematics is disliked by most students--or that it is the least favorite school subject. But in the elementary school

It is true that in some surveys a significant proportion of pupils rated mathematics as the least liked of their school subjects. But it is equally true that in these surveys approximately the same proportion of pupils (at least 20%) cited mathematics as the best-liked or the second best-liked subject. (Suydam & Weaver, 1970, p.4)

Similarly, in a recent study by Ernest et al. (1975, 1976), students in Grades 2 through 12 were asked to rank mathematics, English, science, and social studies. Mathematics was liked best by 30% of the boys and 29% of the girls, and liked least by 27% of the boys and 29% of the girls.

Suydam and Weaver (1970) reported that their review of research indicated that "boys seem to prefer mathematics slightly more than do girls, especially toward the upper elementary-school grades" (p. 4). Ernest et al. (1975) found however, that mathematics was the only subject in which no sex difference in preference was observed.

In two long-term studies involving data from the mid-1970s, Anttonen (1968) found that mean attitude scores declined between Grades 5 and 6 and Grades 11 and 12, while Crosswhite (1972), examining measures of attitude,

self-concept, and anxiety as one phase of the National Longitudinal Study of Mathematical Abilities (NLSMA), reported that student attitudes toward mathematics peaked near the beginning of junior high school. Aiken (1970) concluded from his thorough review of research that children's attitudes appear to become increasingly less positive as they progress through school; more recent studies continue to support this conclusion.

Mathematics educators and teachers believe that the affective component of learning is important: If children are interested in and enjoy mathematics, they will learn it better. However, research indicates that positive or negative attitudes toward mathematics appear to have only a slight influence on achievement in mathematics. And it may well be that achieving well in mathematics has the effect of making attitudes more positive.

Suydam (1975) summarized results of 12 studies reported between 1962 and 1973. When significant correlations were found between attitude and achievement, they generally ranged between .20 and .40; that is, no more than 4% to 16% of the variance in achievement could be accounted for by attitudes. There is, however, a rough balance between studies in which no significant differences are reported and those in which a significant correlation was found. There is not, however, any differing pattern across the years.

Teachers are widely believed to be prime determiners of a student's attitude and performance. Smith (1974), for instance, reported that students' perceptions of teachers were significantly correlated with mathematical growth in Grades 4 through 6. Rosenbloom et al. (1966) found that teaching effectiveness contributed significantly to the attitude and perceptions of pupils concerning their teachers and their methods, the school, text materials (SMSG), and the class as a group. Good, Biddle, and Brophy (1975) presented correlational data showing that some teachers consistently outperform others in affecting student learning or attitudes. However,

there is also some evidence (e.g., Kester, 1969) that attitudes were not significantly affected by teacher expectations or behaviors.

It is also believed that parents determine the child's initial attitudes and affect their child's achievement. Poffenberger and Norton (1959) stated that attitude toward mathematics is a cumulative phenomenon caused by one experience building on another. Attitudes, they believe, are developed in the home and carried to the school; self-concepts in regard to mathematical ability are well established in the early school years, and it is difficult for even the best teacher to change them. Parents influence the child by their expectancy level, by their degree of encouragement, and by their own attitudes toward mathematics. Many parents expect above-average work in general, but are satisfied with only average work in mathematics. Many students report that their parents say, "I'm poor in math," and feel that this gives them sanction for being poor students.

Green (1978) pointed out that cultural differences can also affect students' reactions to mathematics. She reported the avoidance of mathematics by American Indian students, seemingly related to the

widespread, operative fear and loathing of math; the widespread fear of failure in math; and the widespread lack of basic math skills needed to do other academic tasks. (p. 2)

Traditionally raised Indian students appeared to reject mathematics because of the images it evoked, their anxiety about abstract reasoning, and their inability to see its relevance. Green noted the parallels in avoidance behaviors with those of women.

Reviews of research on attitudes (Aiken, 1970, 1972; Knaupp, 1973; Neale, 1969; Suydam, 1975; Suydam & Weaver, 1970, 1975) have confirmed two other generalizations:

1. Relatively definite attitudes about mathematics have been developed by the time children reach the intermediate grades (approximately age 9).
2. There is no evidence that the content or the curriculum per se has particularly influenced attitudes. Evidence has frequently been cited that students like a particular course or program--but comparative data (do they like one course more than another similar one) are not feasible to obtain.

Self-concept. How children feel about themselves and their concepts of themselves while doing mathematics are important components of the affective domain. If certain feelings are experienced over a period of time, they can lead to a particular self-image on the part of children, which can influence what they expect of themselves and can affect their performance.

Self-concept and achievement in mathematics were found to be significantly related in studies by Bachman (1969), Hayes (1968), Koch (1972), Messer (1972), Moore (1972), and Stillwell (1966). Moore noted that, while it may be concluded that self-concepts and attitudes toward mathematics influence achievement in mathematics, it is also reasonable to infer a reciprocal cause-effect relationship between these variables.

Correlations between self-esteem and achievement may be more positive for girls than for boys. Primavera et al. (1974) suggested that the school plays a greater role in affecting girls' self-esteem because it is a major source of approval and praise for girls, whereas boys can seek approval through athletics and other activities.

In almost an equivalent number of studies, no significant relationship between self-concept and achievement has been found (e.g., Birr, 1969; Hunter, 1974; Phelan, 1974; Zander, 1973). Thus, no definitive conclusion regarding the relationship between achievement and self-concept can be drawn from research. The perception that it is a vital factor remains, however,

and many programs and teachers consciously attempt to face the need to develop more positive self-concepts, especially for minority-group students, girls, and low achievers.

Sex differences. Among other student characteristics of increasing concern is that of sex differences. When sex has been incorporated as a factor in the design of a mathematics education study, there is a pattern in the findings across grade levels. As reported in a review by Fennema (1974):

No significant differences between boys' and girls' mathematics achievement were found [in 38 studies examined] before boys and girls entered elementary school or during early elementary years. In upper elementary and early high school years significant differences were not always apparent. However, when significant differences did appear they were more apt to be in the boys' favor when higher-level cognitive tasks were being measured and in the girls' favor when lower-level cognitive tasks were being measured. No conclusion can be reached concerning high school learners. (from abstract, p. 113)

This confirms findings of other reviewers (e.g., Suydam & Weaver, 1970, 1975). Data from the National Assessment of Educational Progress (NAEP) on mathematics indicate that "neither sex has a clear advantage in computational ability since results for males and females varied at the different age levels" (NAEP, 1975a, p. 35).

Fennema and Sherman (1976) discussed variables hypothesized to be related to achievement of women in general and to mathematics learning and studying in particular. They considered verbal ability, spatial visualization ability, confidence in learning mathematics, mathematics as a male domain, attitude toward success in mathematics, perceived attitudes of parents and teachers toward one as a learner of mathematics, usefulness of mathematics, and motivation. Three conclusions were drawn: (1) sex-related differences in mathematics achievement are not universal; (2) many fewer females than males study mathematics in eleventh and twelfth grades; and (3) the relationship between cognitive factors and differential learning of mathematics by

the sexes is unclear. Sex-typing in mathematics and science has been noted by several researchers, and many have documented the fact that there are fewer and fewer women in mathematics as age level increases from junior high school through college.

Whether different provisions for instruction for males and females should be made is another question entirely. Fox (1975) has collected evidence on this; it appears that even when special classes are provided for girls, the attrition rate is high. Difference in aptitude and achievement seem to vary more with individuals than by sex. Societal expectations, changing dramatically in the past ten years in terms of women's roles, as yet seem to have little influence at the secondary-school level, where peer relationships are so important.

Socioeconomic status. There has been so much written on the effect of socioeconomic differences that it seems pointless to belabor the point here. When socioeconomic status has been incorporated as a factor in designing other mathematics education studies, students from high socioeconomic levels tend to achieve better than students from low socioeconomic levels (e.g., see Dunkley, 1965; Johnson, 1970; Montague, 1964; Passy, 1964; Unkel, 1966; NAEP, 1975a). When racial and ethnic minorities have been considered specifically, the members of these groups in general achieve as well as or less well than members of the majority (white/Caucasian) group: They rarely achieve better (e.g., see Asbury, 1970; Castaneda, 1968; Centrone, 1973).

Assessment data from various states parallel the Utah (Ellison et al., 1975) finding: Socioeconomic status was highly related to mathematics achievement, with students from high-income neighborhoods generally having higher mathematical scores. Freda (1976), in his study of 244 California schools, reported that education of parents and income of fathers were the two "input characteristics" most highly correlated with mathematics achievement. In a study conducted in Philadelphia (Summers & Wolfe, 1975),

low income, black, or low ability students were found to be "handicapped in terms of expected achievement growth" (p. 11). They identified correlates of achievement for such students, including smaller schools, especially for black elementary school students and low achievers in senior high school. They also noted that

. . . the more heterogeneous a student body is in terms of race and achievement, the better a student body will fare in terms of race and achievement (p. 18).

The conclusion of the controversial Coleman Report (1966), despite subsequent criticism of its validity, has been widely cited--that, in general, schools exert very little influence on the achievement of children independent of their family background and social context. However, the achievement of minority pupils "depends more on the schools than does the achievement of majority pupils" (p. 22). The Coleman Report has been severely criticized for inappropriate and inadequate statistical procedures, measures of student achievement, sampling, and control of variables.

In a reassessment of Coleman's data to consider comparative contributions of verbal, non-verbal, reading, mathematical, and general informational achievement, Boardman et al. (1973) reported that both the home and the school were important for all achievements, especially verbal and general informational. The explanatory variables considered, however, appeared to be less important for mathematics than for other achievement. Bredemeier (1967) also analyzed data from Coleman and data from Project Talent. The differential achievement of secondary-school students in mathematics had low correlations with any measured characteristics of the schools they attend, and only slightly higher correlations with family background.

In yet another reassessment of the data from Coleman and from Project Talent, Jencks and Brown (1975) reported that

. . . high-school quality accounts for only 1.0 to 3.4 percent of the variance in twelfth-grade test scores, 0.2 to 2.4 percent of the variance in educational attainment, and 2.5 to 4.8 percent of the variance in occupational status and career plans. This means that even if we knew how to eliminate all disparities in high-school quality, which we clearly do not, we could reduce the standard deviations of these outcomes by only one or two percent. (p. 32)

They concluded, on the basis of their analysis, that

Some high schools are more effective than others in raising test scores. Nevertheless, the gains are never large relative to the variance of initial scores, and schools that boost performance on one test are not especially likely to boost performance on other tests. Moreover, high-school characteristics such as social composition, per-pupil expenditure, teacher training, teacher experience, and class size had no consistent impact on cognitive growth between ninth and twelfth grades. . . . Our data tell us nothing about what methods might be most effective. They tell us only that more money, more graduate courses for teachers, smaller classes, socioeconomic desegregation, and other traditional remedies are unlikely to have much effect. (p. 321)

They cautioned also that legislatures and school boards who want to hold high schools accountable for their students' achievement should be "extremely careful to specify the outcomes that really interest them" (p. 321).

Karweit (1976) questioned studies which had led to conclusions about a strong relationship between number of years of schooling and achievement. Her concerns stressed caution in terms of sampling and statistical procedures. In yet another approach, Edmonds et al. (1977) analyzed data from Detroit schools in an effort to identify those which were instructionally effective for poor or minority children. They reported that "family background neither causes nor precludes elementary school instructional effectiveness" (p. 5).

In regard to the "cumulative deficit phenomenon," Deutsch (1965) noted that when one adds a number of years of school experience to a poor environment, plus minority group status, what seems to result are children who appear to be less capable of handling intellectual and linguistic tasks. A consequence of retarded language development is the child's slower and less complete transition from concrete to abstract modes of thought.

In a somewhat dated discussion in the early 1960s, Ausubel (1964) listed some effects of cultural differences, citing low achievers as having: poor perceptual discrimination skills; an inability to use adults as sources of information, correction, and reality testing, and as instruments for satisfying curiosity; an impoverished language-symbolic system; and a paucity of information, concepts, and related propositions. He proposed that teachers of these children must emphasize:

1. The selection of initial learning material geared to the learner's existing state of readiness.
2. Mastery and consolidation of all ongoing learning tasks before new tasks are introduced so as to provide the necessary foundation for successful sequential learning and to prevent unreadiness for future learning tasks.
3. The use of structural learning materials optimally organized to facilitate efficient sequential learning. (p. 27)

In summary, the evidence seems to indicate that SES and achievement in mathematics are correlated, but that the school alone can have little hope of narrowing the achievement differential, nor are the solutions simple.

The educationally disadvantaged student. The educationally disadvantaged have been a source of concern since long before 1965, when federal attention was focused on them. Students labeled slow learners and low achievers, as well as those who are handicapped physically, mentally or otherwise, began to receive relatively more attention in the early 1960s. For example the

NCTM formed a Committee on Mathematics for the Non-College Bound in 1963 that became primarily concerned with the low achiever. SMSG experimented with materials for the culturally disadvantaged (Leiderman et al., 1966) and with a slow-paced beginning algebra course designed for a two-year time span rather than the typical one-year course (Herriot, 1968). The Thirty-fifth NCTM Yearbook (Lowry, 1972) also focused attention on the slow learner.

With the passage of ESEA Title I in 1965, the attention of education shifted to the elementary school. The assumption that starting the child in school "correctly" would yield achievement benefits in later years interacted with the compensatory education thrust of providing for early success in school by enriching the child's environment. Evans (1971) documents the character of many such programs, including Head Start and Follow Through, that were designed to facilitate success in the early elementary school. He noted that they neither emphasize conceptual objectives nor refer specifically to mathematics. Perhaps the implications of compensatory education programs for mathematics may be realized in terms of more general characteristics, such as parental involvement and behavior analysis.

Osborne and Nibbelink (1975) identified some of the many evaluative studies of compensatory programs, noting that those cognitively oriented programs that carefully control the structure of the learning environment and activities appear most successful. They evaluated one such program, Distar (see page 16), which evolved from the preschool program by Bereiter and Englemann. It encompasses a limited number of objectives, most oriented toward the acquisition of verbal language skills. An extensive task analysis led to development of lesson formats to teach number-oriented skills, with few geometrical concepts considered.

Pattern drill and practice procedures typical of the teaching of foreign language (based on behavior modification) are among those methods used in teaching mathematics. The hierarchy of the skills and concepts of arithmetic provides an efficient structure for the language-oriented teaching methodology. Arithmetic is a special form of language. The child develops this language by small, incremental steps. (Osborne & Nibbelink, 1975, p. 282)

The teacher is provided with a script on what to say and directions on what to do; expected pupil responses are cited. Control of pacing, cadence for oral counting, and reinforcement techniques are prescribed. It is a rigid, controlled approach. Osborne and Nibbelink note that

At many points in the program, shrewd insight into a difficulty of learning is exhibited by the nature of the small incremental step designed as treatment for the difficulty. Linguistic and perceptual cues are carefully programmed into the instructional activities. Some of these are extinguished or withdrawn later. (1975, p. 284)

They cited several other positive features, noting also that the program has been found "successful in helping children acquire skills in arithmetic."

They then state:

The DISTAR program may have some severe limitations. Several strong assumptions implicit to the DISTAR approach need to be tested. For instance, DISTAR may be too efficient. Children have little opportunity to "monkey around" in a mathematical context. The role of mathematical play in developing intuition is not known precisely. Second, the perceptual models used for developing concepts are limited in type and in use. . . . Finally, mathematics appears to be a language. The doing of mathematics becomes a problem of translation, of decoding and encoding. . . . Is this too limited a perspective to establish early in a child's experience? (1975, p. 284)

A later refinement of Distar is the University of Oregon Direct Instructional Model, developed by Engelmann and Becker and used in many Follow Through programs. Small-group, face-to-face instruction by a teacher using carefully sequenced, daily lessons are emphasized. Becker (1977) noted:

Four assumptions underlie the model. First, all children can be taught, regardless of their development readiness or background. . . . Second, learning the basic skills, including logical procedures, is central to intelligent behavior and should be essential to any compensatory education program. Third, disadvantaged youngsters tend to be behind other students in skills needed to succeed in school as they are now structured. Fourth, in order to "catch up," the disadvantaged must be taught more in the time available than advantaged children. (p. 521)

The model contains seven essential teaching components:

1. The teaching of general cases is emphasized.
2. The number of classroom instructors is increased.
3. The daily program is carefully structured.
4. Rapid-paced, teacher-directed, small-group instruction is employed.
5. Positive approaches are used to secure and maintain student attention, reinforce responses, and identify mistakes.
6. Teaching staffs are carefully trained and supervised to ensure that appropriate skills are provided and maintained.
7. Student progress is monitored by biweekly criterion-references tests and reports of lessons taught.

The mathematical component of the program operates as follows:

Arithmetic I teaches basic addition and subtraction first through a problem-solving approach; next, to prepare for more elaborate problems, children speed up adding and subtracting by memorizing number facts. In Arithmetic II, the students are introduced to multiplication and fractions; addition and subtraction skills are extended, and a variety of measurement concepts involving time, money, length, and weight are taught. The students are also taught how to work story problems, and how to derive unknown facts from those already known. In Arithmetic III, the students are taught algebra, factoring, and division in addition to traditional computation. (Becker, 1977, p. 522)

Evaluations have indicated that the Direct Instruction Model shows significant program gains in measures of basic skills, cognitive skills, and positive affect (Abt Associates, 1976, 1977). The 20 communities that have used the Direct Instruction Model included a cross-section of lower socioeconomic groups: rural and inner-city blacks, rural whites, Mexican-Americans, Spanish-Americans, Native Americans, and students from a variety of ethnically mixed communities. On the arithmetic portion of the

Wide-Range Achievement Test, the Direct Instructional Model group advanced from the 19th to the 54th percentile. On the arithmetic section of the Metropolitan Achievement tests, students scored at the 48th percentile.

Stallings (1975), reporting on relationships between first- and third-grade instructional practices and child outcomes in the seven Follow Through programs, reported that results showed that time spent in mathematics activities, and high rates of drill, of practice, and of praise contributed to higher scores. Kennedy (1978) reported that almost every model demonstrated lower effects in New York City and Philadelphia than were shown in other locations.

The Direct Instruction model was a notable exception to that rule, and did not display any diminution of effects in the large cities. . . . [which] may mean either that the sponsor was more assertive or that the model was more easily learned. (p. 10)

Criticisms of the Direct Instruction Model have been many; Becker (1977) countered several of these:

1. Scripted presentation of lessons: may restrict a teacher's initiative, but provides "quality control" and serves as in-service instruction.
2. Small-group instruction: "pushes" children, but is more efficient than either one-to-one or large-group instruction, as well as providing a setting in which the necessary repetitions practice can be made fun and challenging.
3. Signals for group response: fosters an authoritarian role for the teacher and may foster rote learning; however, a group needs a leader to imitate, while the data do not support the rote-learning fear.

He also indicated his belief that "schools are designed to teach middle-class children and need to be redesigned for teaching all students. . . . schools systematically fail to provide instruction in the building blocks crucial to intelligent functioning, namely, words and their

referents. Children from homes where there is strong adult support for refining the use of language are more likely to succeed in school than those from homes with less adult-child contact and adults with less education (Coleman, 1975; Freeberg and Payne, 1967; Glass, 1973)" (p. 533).

In another review of evaluations of compensatory education programs at national, state, local, and program levels, Stickney (1976) found "very little evidence that compensatory education has been able to arrest the accumulative achievement deficit that exists between advantaged and disadvantaged pupils." He suggests that "as long as schools remain marginal institutions they are unlikely to compensate for environmentally determined differences in academic achievement" (p. 2088).

The television program "Sesame Street," designed to narrow the gap in the backgrounds of children as they entered school, was effective, according to evaluations--but effective for all children, regardless of racial, ethnic, or socioeconomic background. Only if disadvantaged children watched the program a great deal did they surpass the middle-class children who watched only a little (Bogatz & Ball, 1971).

From a review of reports on summer compensatory education programs, Austin et al. (1972) concluded:

1. The programs in mathematics, reading, and language-communication have generally shown modest achievement gains. However, no randomly formed control groups were used, and no data were found to demonstrate whether these gains persist over time.
2. Students reported an increased desire to attend school and learn the cognitive skills. However, no data were reported to indicate if behavior changes were observable during the school year.
3. Few objective measures have been used to measure the possible range of student accomplishment. (p. 179)

The concern for the educationally disadvantaged student encompasses more than simply low achievers; the handicapped have also been a major concern. In the 1950s, emphasis was placed on special education for mentally retarded and other handicapped students, since their needs were not being adequately served in the regular classroom. In the 1970s, however, just as most schools had made provision for such classes, a move in the opposite direction occurred: "Mainstreaming" was advocated by federal planners in order to ensure equality of education. It also seemed apparent that both "special" and other students derive social and psychological benefit from interaction.

From a review of a large number of studies on the academically and environmentally "disadvantaged," Suydam and Weaver (1971) concluded:

- a. The disadvantaged, as well as all other pupils profit from special attention from the teacher, the content of the program, the instructional materials, and the organization for instruction.
- b. The mathematical characteristics which distinguish disadvantaged from advantaged pupils appear to exist in degree rather than kind. That is to say, disadvantaged and advantaged pupils have similar abilities and skills, but differ in depth or level of attainment.
- c. Rate of learning is but one variable to be considered in providing effective instruction for slower learners. Methods of instruction also must be adapted to these pupils.
- d. Social relevance appears to be more crucial to consider in the case of disadvantaged students; however, little research has attended to this topic.
- e. The degree of meaning (in the mathematical sense) which is optimal for disadvantaged students is an unknown factor. While there is some evidence that "discovery" approaches are not as effective as rule approaches with low achievers, it may merely be that more-closely-guided discovery and lower levels of meaning are appropriate for these groups.

- f. Active physical involvement with manipulative materials, which is believed to be important for all children, may be even more so for the disadvantaged.
- g. Pupils who are disadvantaged mathematically may also be disadvantaged in other factors which are related to their mathematical learning (e.g., reading ability). Such things must be taken into account in planning the curriculum for the disadvantaged child.
- h. It does little good to report that special programs for disadvantaged students are effective without also reporting in detail the specific nature of those programs. More evidence on "ideas which work," as well as research, is needed.
- i. Groups of disadvantaged pupils are not all disadvantaged in the same way. There is as much need to individualize instruction for disadvantaged students as for other groups of students. (p. 47-48)

The talented or gifted student. Acceleration, ability grouping, special courses, and enrichment have always been the obvious means for coping with the talented. However, two categories of programs, acceleration and enrichment, are the primary alternatives for serving the interests of the exceptionally gifted in mathematics. Research provides no significant evidence that one of these methods is to be preferred over the other. However, the constraints imposed by the local school situation and the sort of talent found in the mathematics teaching staff may well determine what is possible at the local school level. Data on the percentage of schools using each varies from survey to survey; in general, however, it appears that special courses are the most frequently used procedures and acceleration the least used. For the junior high school, Begle (1976) reviewed 42 research reports dealing with acceleration for talented students and concluded that acceleration was preferable to enrichment at that level. Studies at all levels indicate that care should be taken to select the option that is most appropriate for the individual.

Special curricula for the talented in mathematics have been created to serve the upper tenth of the student population. The Secondary School Mathematics Curriculum Improvement Study (SSMCIS) at Columbia University and the Comprehensive School Mathematics Project (CSMP) at CEMREL both developed curricula for the secondary-school student of exceptional talent, representing attempts to realize the curriculum advocated in the Cambridge Conference Report (1963). Both of these curricula can be successful in a school with a sufficiently large population of talented students and a staff with the mathematical capability for teaching the curricula.

It should be noted that mathematical ability consists of a variety of factors, and mathematics achievement is influenced by a variety of factors. The background of knowledge and expectation that the student brings to school appear to be of utmost importance: Teachers and schools cannot hope to compensate, and thus narrow the achievement gap, without major upheavals in the non-school environment. To expect schools to be accountable, and not parents and society, is posing an impossible goal, whose outcome will be labeled a failure. Expectations for schools must be reevaluated, so that the gap between what-can-be-done and what-should-be-attained is narrowed. The various clientele of the schools need to think carefully about the aims of the school mathematics program.

Instructional Materials

Textbooks, supplemented by workbooks and other materials for seatwork or homework, are heavily relied upon in mathematics teaching. But other types of materials are also endorsed for use in mathematics classrooms; an NCTM Yearbook and several supplementary publications have tried to focus attention beyond the textbook.

Textbooks and other print materials. The textbook is the primary determinant of mathematical curricula throughout schools in this country.

Within most classrooms, a single textbook is used with all students, rather than referring to multiple textbooks or varying text use by group or individual needs (e.g., see Graeber et al., 1977, p. 19). There is rather firm adherence to "covering the material" in the text, although sections which teachers do not consider important (and which may not be included on standardized tests) may be ignored. Elementary-school geometry has suffered this fate for years. Begle (1973) reported that different patterns of achievement were associated with the use of different textbooks.

Graeber et al. (1977), from a survey of 1,343 teachers in 3 Eastern states, reported that the 10 most frequently used textbooks accounted for about 90% of the first-grade classes and about 67% of the classes at Grades 3, 5, and 7.

A report (EPIE, 1976a) on an unpublished study of most-used instructional materials stated:

The ten most used materials in mathematics are clearly traditional programs, all quite similar to each other in terms of instructional design. They are also traditional in terms of the way they were developed. . . . If we look at the first 32 mathematics materials listed, only one program is the result of nontraditional development and this development was federally funded. This material is rank-ordered 24th. Of the remaining 31 materials, at best two could be considered to have even a modicum of an R&D base . . . built upon an empirical data base, as opposed to conventional wisdom. (p. 1).

Among other highlights of the survey of 12,389 teachers, including 4,455 mathematics teachers (K-12) were (EPIE, 1976b):

- Instructional materials, print and nonprint, are used during 90 to 95 per cent of all K-12 classroom instructional time. Schools spend about 1 per cent of their budgets on these materials . . . (p. 1).
- Teachers tend to be unclear about how good a "fit" there may or may not be among their teaching, the materials they are using, and the needs and abilities of their students . . . (p. 2).

Many textbook analyses have been reported, spanning the years (e.g., Burns, 1960; Buchalter, 1969; Clason, 1969; Dahle, 1970; Folsom, 1960; Kahn, 1974; Maura, 1957; Neatrour, 1969). Generalizations from such studies include:

- Low-level cognitive processes--knowledge and comprehension--are used far more frequently than high-level processes.
- There is considerable agreement on grade placement, sequence, and presentation of basic topics.
- There is wide variance in the total number of concepts and the amount of space devoted to the various topics.
- Relationships are found between textbook emphases and social or psychological trends.
- An almost total emphasis on computational skill in elementary school textbooks is apparent.

Brown (1974) conducted an in-depth study on the use of textbooks made by teachers and students in geometry and algebra 2. Very heavy dependence on the textbook was found: "Teachers followed the textbook very closely with regard to content selection and sequencing. The major objective of observed lessons tended to be completion of the exercises presented at the end of the section of the textbook under discussion" (p. 5795). Teachers made little use of special features--historical and bibliographic information and enrichment exercises. They rarely presented topics not in the textbook. Typically, they progressed through the text, section by section. Brown concluded that, for the teachers and classes in the study, mathematics did not extend beyond that which was presented in the textbook: "The subject was resolved into a sterile sequence of homework/discussion/new homework" (p. 5796).

Programmed instruction. The foremost claim for programmed instruction was that it would allow each pupil to progress at his or her own rate. Some studies ascertained the feasibility of using programmed instruction to teach specific content. When compared with conventional instruction, the results

were equivocal (Suydam, 1972). It was evident that programmed materials were most useful when used to supplement, rather than replace the teacher (Lackner, 1967). In his review, Zoll (1969) concluded: "It is not clear from these [35] studies that the strongest single claim for the use of programmed instruction, that each individual learns at his own rate, has been supported" (pp. 107-108). Bobier (1964) noted that low-achieving students of limited ability were not sufficiently motivated to use programmed textbooks independently. For many teachers, it became apparent that programmed instruction was not a panacea. Most could probably agree with the conclusion of Jamison et al. (1974): " . . . PI is generally as effective as TI [traditional instruction] and may result in decreasing the amount of time required for a student to achieve specific educational goals" (p. 41).

Nevertheless, teaching machines gather dust, and programmed instruction is rarely discussed. However, the idea is still apparent in computer-assisted instruction programs and in self-paced programs.

Manipulative and other materials. Once it was not uncommon for a teacher to make or collect inexpensive instructional materials for use in the classroom. Then, with NDEA in 1958, federal funds for a wide variety of materials began to be available. Evidence from a range of sources indicates that this money was not always spent frugally and extensive care in selecting appropriate educational materials was not always taken. Part of the reason for this stemmed from the fact that too frequently money became available at short notice and its use was not accounted for specifically. As budgets have tightened over the past several years, teachers have at times resorted to the plea of "unavailability of funds" to account for their failure to use materials.

In the review of research on the use of materials in elementary school mathematics (K-8) conducted by Suydam and Higgins (1976), it was reported:

1. In almost half of the considered studies, students having instruction in which manipulative materials were used scored significantly higher on achievement tests than students who had instruction in which manipulative materials were not used. In almost the same number of studies, the two groups scored much the same; few instances were found in which the group not using materials scored higher. Thus, lessons using manipulative materials have a higher probability of producing greater mathematics achievement than do non-manipulative lessons.
2. Only 3 of 28 findings favored the use of symbols alone; only one study favored pictorial treatments used alone. In 7 instances, use of manipulative materials was favored over sequences in which manipulative materials were not used. In 9 instances, use of manipulative materials and pictorial representations resulted in higher achievement than use of symbols alone. The concrete materials thus appeared to play an important role.
3. Research in which the number of embodiments for a mathematical idea has been the focus resulted in no significant differences in achievement in 3 of 4 studies.
4. In 3 of 8 studies, manipulation of materials by students was favored over having students watch the teacher demonstrate with materials. In 4 other studies, no significant differences were found. It appears that individual manipulation by the learner is not the only way children learn; it can be effective to watch the teacher demonstrate.
5. Across a variety of mathematical topics, studies at every grade level support the importance of the use of manipulative materials. Little evidence was found that manipulative materials are effective only at lower grade levels.

6. The use of materials appears to be as effective at one achievement level as at another--that is, high achievers profit from the use of materials as much as low achievers.
7. The use of materials appears to be as effective at one ability level as at another--that is, those of high ability profit from the use of materials as much as those of low ability do.
8. Although the data are sparse, the use of materials appears to be at least as effective at one socioeconomic level as at another.

The extent to which materials are used has been considered in several surveys. Johnston (1962) found that few teachers in Grades 1 through 8 used any material other than the textbook. Green (1970) reported that first-grade teachers used more materials and used materials more frequently than sixth-grade teachers. Haladyna (1975), in a study with 4,400 Oregon teachers, also found that with "the use of manipulatives the tendency was for moderate to frequent use in the primary grades to a minimal use at the intermediate, high school, and junior high school levels" (p.8). Both the NACOME (1975) and Graeber et al. (1977) surveys concurred with this finding. In another report on Oregon projects which focused on various materials, Thomas (1975) found that in no instance were either manipulative materials or games the basis of a significant percentage of programs. However, teachers' attitudes toward the use of both was very positive; the use of manipulative materials was not as favorably viewed by students, however.

The Developing Mathematical Processes program developed at the University of Wisconsin-Madison integrates a variety of materials in its measurement-oriented approach. Necessary materials are available in kits; nevertheless, some educators have noted that many teachers do not make use of these materials. The same response shows up in connection with materials

provided with a variety of other programs. Thus Schoen (1977), in his review of research on self-paced instruction, confirmed this:

There is consistent evidence that the use of various media and supplementary teaching materials increased the effectiveness of SPI. There is also consistent evidence that media and materials in a typical SPI program have been restricted to printed and audio materials. In addition, the various media and supplementary materials often have not been used even when available. (p. 213)

The NACOME Report (1975) indicated that

In spite of the recent publicity and emphasis it is not at all clear that manipulative materials are widely used. For instance, 37 percent of the elementary school teachers in the NCTM survey had never used the mathematics laboratory, and ten percent had never used manipulative materials at all (in grades 2 and 5). (pp. 62-63)

Additional means must be found to encourage teachers to use materials. But it is also necessary to consider carefully what, when, how, why, and by whom the materials will be used.

There is relatively little evidence on the amount of use of various audiovisual devices. Generally, they are collectively studied as one of a variety of instructional materials. Many reports indicate the availability of equipment such as film and filmstrip projectors, television, and overhead projectors, but their actual use is not yet an everyday occurrence.

Computer-aided instruction. In the early 1960s some schools began to buy or lease computers or computer time, usually first for administrative purposes, and, after a time, for mathematics instruction. The Dartmouth model, funded by NSF, has been extensively copied.

Three modes of computer use have evolved:

1. computer-aided: non-tutorial, problem-solving aid

2. computer-assisted: tutorial instruction with the computer taking a teacher's role
3. computer-managed: courses of study are developed, sequenced, and/or monitored for students, with the computer storing information.

Two large-scale national surveys (Darby et al., 1970; Bukoski & Korotkin, 1976) of computing activities have been conducted (in 1969 and 1974) by the American Institutes for Research. A stratified random sample of 25% of the secondary schools was selected for the second study, plus a sample of the schools participating in the 1970 study; responses were received from 3,643. Since 1970, the fraction of secondary schools reporting some computing activity has steadily increased, from 34.4% in 1970 to 58.2% in 1975. Mathematics classes used the computer most frequently, although the percentage dropped from 46.6% to 43.2%.

The researchers projected:

Though the continued growth of computer-based education seems assured, the specific future of instructional computing is unclear. Based upon the growth over the last five years (1970-1975), it is projected that within the next decade the majority of secondary schools in the country will have some type of instructional computer-based application. . . . it is probable that computer science and problem solving will remain prominent instructional applications through the next decade. (p. 20)

Despite the growth in computing activities, they indicate that the relative costs remained virtually the same.

Rudolph (1972) found that one-third of the 647 Illinois secondary schools she surveyed used computers, with 54% of these using computers for both instruction and administration, and only 5% solely for instruction. Problem solving in mathematics and science, as well as data processing, accounted for over 80% of the time. Bishop (1971) reported that 30% of the secondary schools in the Missouri region offered technically oriented computer-

related courses in their mathematics curriculum; 20% used computer time for enriching and supporting courses.

Moran (1974) reviewed practices and trends. He noted that time-sharing had grown in importance; however, the minicomputer and programmable calculators have had and will continue to have an impact on school use of computing power.

Studies on the effectiveness of the use of computers were reviewed by Kieren (1973) and by Hatfield (1973). In general, the results are equivocal: Higher general achievement is not a foregone outcome of the use of computers, but it does aid in promoting problem-solving achievement. Batch processing appeared to be at least as effective as having direct computer access: The important factor may be experience in writing programs rather than the time it takes to receive computer solutions.

Jamison et al. (1974), surveying some studies using mathematics content, stated:

. . . no uniform conclusions can be drawn about the effectiveness of CAI. At the elementary-school level, CAI is apparently effective as a supplement to regular instruction . . . At the secondary-school level, a conservative conclusion is that CAI is about as effective as TI when it is used as a replacement. It may also result in substantial savings of student time in some cases. (p. 55)

Vinsonhaler and Boss (1972) reviewed seven major studies on drill and practice programs using CAI. They indicated that higher achievement could be anticipated when CAI was used to augment regular instruction. They noted that

There are indications that the effects obtained with CAI might be obtained through less expensive means. For example, one of the studies reported by Suppes and Morningstar (1969) suggests that an additional 30 minutes of ordinary classroom drill and practice can accomplish the same results as a 15-minute CAI program. (p. 31)

In a study concerned with students' reactions, Hess and Tenezakis (1973) reported that both students who had used a CAI drill-and-practice program in basic arithmetic for one or two years and non-CAI students regarded the computer more favorably than they regarded their teachers or their textbooks. Such findings should be the subject of more extensive study.

Calculators. The hand-held calculator has been on the market since the early 1970s. In 1975, the cost of calculators dropped sharply, and as a result "for millions of people, everyday arithmetic will never be the same" (McWhorter, 1976). Desk calculators had been used in some secondary-school mathematics classrooms, but their use was largely restricted to low achievers, and they generated little excitement. Calculators that could be readily available for each and every student created a changed perspective.

A position statement of the NCTM (1974) reflects the immediate concern of mathematics leaders:

Mathematics teachers should recognize the potential contribution of this calculator as a valuable instructional aid. In the classroom, the mini-calculator should be used in imaginative ways to reinforce learning and to motivate the learner as he becomes proficient in mathematics.

Other groups throughout the country also recognized the potential of the calculator. The NACOME Report (1975) severely questioned the strong trend to emphasize computation; the case for decreasing the virtually exclusive emphasis on computational skills was seen as stronger than ever before because of the impending universal availability of calculators. They noted that many low-achieving students have been condemned

to a succession of general mathematics courses that begin with and seldom progress beyond drill in arithmetic skills. Providing these students with calculators has the potential to open a rich new supply of important mathematical ideas for these students . . . at the same time breaking down self-defeating negative attitudes acquired through years of arithmetic failure. (pp. 41-42)

Therefore, they recommended use of calculators "beginning no later than the end of the eighth grade," with the student permitted to use the calculator during all mathematical work including tests. The development of instructional materials and curricular revision or reorganization "in light of the increasing significance of computers and calculators" were also recommended.

The Euclid Conference (NIE, 1975) participants also indicated concern with the effect of calculators on the curriculum, stressing the need for developing new sequences of instruction. The National Science Foundation, concerned about the potential impact of the calculator, funded a critical analysis (Suydam, 1976). All existing literature was studied, and a survey conducted to ascertain the arguments for and against use of calculators and the ways in which calculators should be used. Frequently cited reasons for using calculators included: aid in computation; facilitation of understanding and concept development; lessening of the need for memorization; help in problem solving; motivation; aid in exploring, understanding, and learning algorithmic processes--and the fact that they exist, and are appearing in the hands of increasing numbers of students. Their usefulness in helping low achievers to succeed was also frequently cited.

The most frequently cited reasons for not using calculators were that: They could be used as substitutes for developing computational skills; they are not available to all; and they may give a false impression of what mathematics is. The first concern was expressed most frequently by parents and other members of the public; few educators, however, believed that children should use calculators in place of learning basic mathematical skills.

Analysis of studies comparing calculator and non-calculator groups indicated that in about half the instances, the calculator group achieved significantly higher on paper-and-pencil tests (with which the calculator was not used); no significant differences were found in almost half of the

studies; in only a few instances was achievement significantly higher for the non-calculator group (Suydam, 1978).

A conference on the uses of hand-held calculators in education was held in June 1976 by NIE and NSF to produce a planning document "that will provide a well-defined framework for future research and development efforts." The participants noted:

These small, portable, and inexpensive machines have the potential for replacing the paper and pencil calculations that have been the major (and often the sole) component of elementary school arithmetic. (p. 2)

Educators are faced with a dilemma. Their experience and instincts tell them to research, test, and proceed with caution. Yet calculator technology is progressing rapidly, and marketing pressures are great. The evolutionary pace traditionally associated with curriculum change is too slow to fit the present situation. (p. 3)

How extensively the calculator will influence the mathematics curriculum is unclear. Conflict is obvious between those who see computational skills as the most vital task for mathematics teachers and those who see the calculator allowing a change in direction--a change feasible for the first time in history. In the past 3 years, opinions have changed, and the calculator is being used with increasing frequency, but the curriculum has not changed noticeably. Other technological developments are on the near horizon. The dividing line between calculators and computers is already tenuous; existing calculators have the computing power of computers of 20 years ago. Interaction between students and machines will be increasingly feasible.

Federal Funding Impact

Beginning in 1968, the amount of emphasis placed on evaluation of federally funded projects has increased. Reports from those receiving federal funds

indicate that they felt the projects had an impact. Thus McDaniel (1973) indicated that teachers and supervisors in 57 secondary schools with 4 or more NDEA Title III projects "observed improvement in teachers and students" as a result of use of NDEA Title III-funded materials and equipment. No data supporting this claim were reported, however.

In several assessments of the impact of funding, findings were at some variance with official statements. Thus DeShields (1973) reported that students in Title I schools performed at significantly lower levels than those not in Title I schools (but who may have been eligible) and Ordonez (1971) reported that pupils in Title I schools had significantly less positive attitudes toward arithmetic. In both instances, however, pre-existing conditions could have been measured, rather than anything resulting from Title I funds.

A Rand Policy Study (McLaughlin, 1975) is perhaps the most widely quoted analysis of a federal program. McLaughlin traced the evaluation requirements of ESEA Title I, noting that, because of political concerns, "framers of Title I purposely left ambiguous parts of the bill that might generate conflict and weaken support" (p. 17).

. . . an implicit decision was made not to set uniform reporting standards, not to require measurement by standardized tests, and not to suggest what the preferred components of "effectiveness" might be. More sophisticated methodological notions, such as the provision of control groups, were rejected as running against the grain of legislative intent. (p. 19)

Consequently, reports for 18,000 LEAs and 50 SEAs for the first 2 years

. . . painted the success of Title I in glowing terms, and suggested that the local school administrators were moving quickly to devise effective compensatory strategies. Title I seemed to be working beyond anyone's highest expectations. (pp. 22-23)

McLaughlin found that evaluation was not being used to aid in decision making about curriculum and instruction nor to determine priorities at any level--local, state, or federal--nor were they used by SEAs or USOE to determine funding approvals. Because of reactions to reports, however,

Federal interest in the results of the mandated reporting scheme ended with the publication of Title I/Year II. There is no evidence that local reporting practices have improved with time [although states are required to turn in reports from time to time]. Reviews undertaken by the American Institutes for Research (AIR) and the Center for Educational Policy Research, Harvard University, found that these evaluations were as unsatisfactory in 1972 as they were in 1966. . . . if one were to rely solely on these required reports in judging the impact of Title I, one would have to conclude that it has been an astonishing success--a conclusion that . . . finds little support in other efforts to evaluate Title I. (p. 23)

McLaughlin believed that LEAs wanted general aid, not categorical aid targeted for disadvantaged children. Both USOE and the SEAs seemed unwilling to destroy good working relationships "over the relatively trivial matter of Title I data collection and evaluation" (p. 25). It was evident that "it is difficult if not impossible to trace the course of Title I dollars through the school system" (p. 40).

It was also noted, however, that experience with other social programs (particularly health care) suggests that social programs may have "high impact or high coverage, but not both," implying that "measurable benefits from large-scale social action programs such as Title I can be expected to be marginal" (p. 40). McLaughlin noted that academic achievement is but one of many objectives of Title I: Therefore, to "conclude on the basis of standardized test scores that Title I is not (or is) 'working' is not justified" (p. 41).

A summary of viewpoints on similar evaluations is provided by Becker (1977):

Field-based experiments can fail for many reasons, as some have suggested. . . . From our present perspective, several reasons can be suggested why such projects seem to fail: some programs are not well-designed initially (Engelmann, 1975); the self-protective bureaucratic structure of school systems resists change (McLaughlin, 1975); the instruments used to measure outcomes reveal changes caused by maturation rather than school instruction (Becker & Engelmann, 1976); some control groups simply make use of children left over after the disadvantaged are placed in special programs and, in doing so, fail to randomly select and assign students (Haney, Note 1). The lesson is clear: when field programs fail, especially if program installation and operation are not monitored, we have only a heyday for speculation. We learn little of value about those details needed to design better schools. (p. 519)

It may be that the most viable effect of Title I, and of other federal funding efforts, was that attention was called to the need for innovation and change. And it also seems apparent that the involvement of teachers is a crucial factor if change is to be effected and effective.

Progress Assessments

The movement toward accountability has resulted in both minimal competency requirements and assessments of achievement in many states. Pipho (1977) reported that 8 states had legislation requiring some form of minimal competency testing for high school or grade-to-grade promotion, 10 states had state board of education rulings, and legislation was pending in 15 states. In some instances, only reading was considered; in most, mathematics and reading were both included; in a few cases, other goals were also considered.

In Virginia, for instance, the General Assembly listed basic skills in reading, communications, and mathematics first in a set of 10 "standards in quality" (Standards, 1976). They enacted legislation mandating the development of minimum competency objectives and tests with which to assess

them, with little interaction with educational agencies in the state. State departments of education and local school districts were given a relatively short period of time to implement the legislative mandate. Educators had no direct role in the decision-making process, nor was the rationale for the decision-making process clear.

Some local school districts across the country are also adopting minimal competency standards; the total is difficult to determine, but known instances total fewer than 50. Denver led the way, with competency tests administered there since 1962.

The changing status of accountability legislation is also being monitored. As of June 1974, 30 states had accountability legislation (Hawthorne, 1974). In regard to the assessments, it should be noted that:

1. There is great variability in the objectives being assessed. For instance, one state included these two objectives for grade 3:
 - Pupils will indicate ability to analyze by constructing a market value continuum on a given set of objects or pictures of objects.
 - Pupils will indicate application is using the addition and multiplication algorithms by applying those rules to solve addition problems through two 7-digit numbers and multiplication problems of 2-digit numbers.

Other states have restricted the objectives to minimal competencies.

2. Both standardized norm-referenced and criterion-referenced tests were used.
3. Reporting procedures vary widely: some states provide a summary, some present data alone, some provide data plus interpretation. Criterion levels, percentages, grade level norms, and a variety of other statistics are provided.

A review of available assessment reports indicated that:

1. The topics with which difficulty (or weakness) were reported can be ranked in this order of frequency: fractions, division, subtraction with regrouping, decimals, geometry, measurement, proof, estimation, statistics and probability, problem solving. This corresponds with information from previous error-analysis studies and studies on difficulties and the need for remediation.
2. Status was reported as "at norm" and "below norm" in approximately an equivalent number of instances; fewer instances of "above norm" were noted.
3. Trends are unclear: in the few instances where data from the same test administered for 2 or 3 years could be checked, improvement was noted on 4 of 5; in the fifth case, scores remained at about the same level. (Suydam & Osborne, 1977, p. 212)

In other state, local, and regional assessments, some comparing data across a period of years, no clear trend can be observed. Roderick (1974) provides an example of one difficulty in assessments across decades. Not all of the items administered to the 1973 students involved content still being taught; thus, many items were transfer items for the 1973 students. It is also apparent from a scan of the items that mastery levels were by no means achieved on many of the items by the 1936, 1951-55, and 1965 pupils, any more than they were achieved by the 1973 students. Where an item was passed by 80% or more of the earlier students, it tended to be an item on which 1973 students also scored high.

Pinpointing the specific content with which a student, a class, or a statewide sample of students has difficulty may help to focus needed attention. But concomitantly we need more knowledge on how to teach such ideas more effectively. Rote teaching could result in increases in paper-and-pencil scores without (a) long-term retention and (b) the ability to apply mathematical ideas in new situations. Limited goals, resulting from a focus on minimal competencies and accountability, could result in a less mathematically literate group than at present.

A Concluding Comment

In the summary section of the review on existing practices, Suydam and Osborne (1977) cited three primary sources of difficulty in policy formation for mathematics education:

1. Educational policy is frequently determined without collecting enough information to allow the process to be rational.
2. Educational policy is frequently constructed without using information that is readily available.
3. The point at which values enter into policy formation, and the effects of the differences in the values held by various groups concerned with the schools, is frequently not recognized in determining the priorities within educational policy (p. 219).

McLaughlin (1976), in studying the process of change, concluded that change has little permanence in the schools if the need for a project or program is based on "entrepreneurial motivation" rather than a perception of a problem needing solution by the potential users. Thus, a development or research effort will fail at the point of implementation of application of the results if discrepancies are not resolved.

III. EFFECTIVE STRATEGIES FOR URBAN STUDENTS

In preparing this report, ERIC documents* pertaining specifically to urban students were re-analyzed to form a list of strategies, procedures, and programs which had been found effective for mathematics instruction. After listing a dozen points, the effort was halted: Redundancy with previously stated comments was apparent. Appearing on the list were such points as: use of manipulative materials, provide practice, praise, use tutors,

* ERIC provides the largest collection of reports on federally funded as well as local projects.

individualize instruction, provide resource teachers, give feedback, use teacher aides, teach in small groups, and involve parents. Such practices, quite obviously, are useful with all children. The precise reasons for their success with urban children could not always be discerned, for, unfortunately, studies are in general characterized by:

1. Lack of specificity about the program and procedures: replication is impossible.
2. Lack of a control group.

Because of the lack of decisive help afforded by individual reports, attention was concentrated on documents, in addition to those cited in the previous section, which attempted to look more carefully at teacher and student variables.

Recommendations for attacking the problem of improving mathematics instruction in urban schools have come from many sources. Rasmussen (1968) queried college and school personnel about guidelines for developing curricula for seventh and eighth graders; they suggested:

1. including urban school teachers on city-wide curriculum committees;
2. designing curriculums which provide for different student activities;
3. designing curriculums which provide for special needs of local school students;
4. emphasizing curriculum materials which require less reading;
5. selecting cooperatively supplementary curriculum materials;
6. placing experienced teachers of mathematics in urban schools; and
7. providing inservice sessions to develop understandings of urban children and to perfect mathematics instruction skills.

Wellisch et al. (1976) identified four key dimensions of a reading and mathematics program each significantly related to achievement gain, independent of student background characteristics:

1. organizational climate, which produced a composite index of administrative leadership and a measure of district-level support for new teachers that predicted math gain;
2. parent and community involvement, which produced an index of parent participation in the classroom that predicted both reading and math gain;
3. reading and mathematics program characteristics, which resulted in three indices that predicted achievement gains: the use of behavioral objectives, the provision of adequate instructional practice, and less frequent use of positive reinforcement; and
4. reading and mathematics resource use, which uncovered a relationship between achievement gain and per-pupil costs for remedial specialists.

Project LONGSTEP (Longitudinal Study of Educational Practices) was undertaken to identify changes in the achievement of students in Grades 1 through 12 that occurred as a result of educational innovation (Chalupsky & Coles, 1976, 1977; Coles & Chalupsky, 1976; Coles et al., 1976; Everett, 1976; ED 132 176; ED 132 178; ED 132 193). Overall project findings showed that dramatic school effects were not associated with intensity of educational innovation. However, different educational approaches did produce meaningful differences in achievement, especially in the early elementary grades. Large gains in arithmetic skills, over and above those expected on the basis of pretest and socioeconomic status, were found to occur in some schools; students enrolled in programs with a more moderate emphasis on innovation and individualization showed the greatest improvement. There was also a tendency for extent of innovation to be related to positive student attitudes toward schooling.

These findings serve as a reminder that educational innovation per se will not necessarily dramatically effect student achievement. Educational quality is not synonymous with innovation or individualization. Individualization of instruction may represent a valuable approach, but the findings suggest that individualization as a program strategy should not be viewed as the final or complete answer.

Manzo (1975) focused on characteristics of children and teachers. He noted that the mathematics student many have "defects, deficiencies, disruptions, and/or differences" in the cognitive abilities required to perform a mathematical problem " (p. 5). Mathematics teachers may be less verbal and less socially responsive than other teachers. Mathematics teachers also may have internalized mathematical reasoning and numerical notation so that they can hardly understand the "primitive" state of most students and many adults. Students tend not to question, but instead to accept mathematics as a closed system. The mathematics word problem poses a barrier to learning because of the new words and notations and the complex language and terse sentences.

Good and Grouws (1975, 1977) related student achievement in fourth-grade mathematics to teachers' use of various strategies and to classroom environmental factors. Six clusters of variables were found to be strongly associated with teacher effectiveness: (1) student initiated behavior; (2) whole-class instruction; (3) general clarity of instruction; (4) a non-evaluative and generally relaxed environment; (5) higher achievement expectations; and (6) classrooms that were relatively free of major behavior disorders. They concluded that successful teaching appeared to be based on a large number of variables that must be present, to at least a minimal extent, rather than on one or two critical factors.

Marcus et al. (1976) indicated that survey data showed that schools in which principals emphasized the importance of selecting basic instructional

materials and made more of the decisions in the instructional area were more likely to show achievement gains in the subject areas of reading and mathematics.

A survey conducted by NAEP (1978) indicated that, in general, 17-year-old students who reported doing the most homework and the least amount of television-watching performed best on the NAEP mathematics assessment. Similarly, Hushak (1973, 1977) found that student study time had a positive impact and time watching television a negative impact on arithmetic computation. The impact on high-level language and arithmetic skills and concepts was smaller and sometimes reversed. Lower-achieving students were more dependent on teachers than higher-achieving students: They had less educated parents and access to fewer other non-school learning resources. They attempted to compensate by studying more and obtaining more help from their less educated parents, but the teacher was their primary source for skilled teaching. Consequently, schools have a greater marginal impact on lower-achieving students. They tend to equalize achievement among all students even when poorer quality resources, in particular teachers, are allocated to lower-achieving students. Greater equalization could be obtained by allocating the higher quality school resources to lower-achieving students.

At a conference held by the School Mathematics Study Group (SMSG, 1970) on "Mathematics Education in the Inner City Schools," several components of programs for urban children were repeated by various speakers:

1. Teachers should have higher expectations for these children than they frequently do.
2. Students should be involved in learning.
3. Parents should be involved with their child's learning.
4. Natural language and language patterns should be used.
5. Increasing the physical comfort of learners is important.

6. Concepts should be "talked through" as an advance organizer before instruction per se.
7. Special times or days should be set aside for individual skill building.
8. Correction and checking of problems should be immediate.
9. The student's self-image should be enhanced.
10. Careful planning should be done to provide "success experiences."
11. Diagnosis, prescription, and individualization are key components of programs.
12. Evaluation in terms of pre-determined goals should be an ongoing aspect of programs.
13. Teacher training must be directed toward work with urban children.
14. Programs must be relevant to the children.

It should also be noted that differing values appear to lie at the base of the problem. Specifically, the middle-class culture places value on education through such means as having books in the home, providing parental examples of using books and of success in education, encouraging verbalization, and imparting stimulation to achieve. Such values are frequently missing among the disadvantaged. Thus a gap is created early.

Studies from a number of sources suggest that children from more advantaged homes have had experiences of greater variety in an organized family setting than do children in disadvantaged homes. By the time the more advantaged children reach school-age, they appear to be better able to work in a group situation, to utilize verbal skills, and to deal with abstract concepts. (Leiderman et al., 1966, p. 2)

Differing values for thinking, reasoning, problem solving and orderly behavior also appear across groups. Few teachers are trained to be aware of such differences and their implications for instruction.

In a concise but strong paper, Trafton and Morgan (1978) discuss three points they believe to be of particular relevance to urban schools:

1. Mathematics needs to be taught; that is, sufficient time must be devoted to mathematics education.
2. Curriculum content needs to be carefully selected.
3. Instructional time and materials must be used purposefully.

Moreover, they point out some considerations for successful programs for low achievers, including administrative commitment, positive perception of a child's capability of learning, careful assessment of achievement, an efficient management system, ongoing teacher training, specific learning objectives, mastery requirements, and cost effectiveness.

IV. SYNTHESIS OF EVIDENCE

Some of the key points from the evidence on existing practices are summarized on the following pages. (Again, it should be noted that much of the evidence cited was drawn from Suydam & Osborne, 1977.) The strength of the evidence for each point varies, but the studies met certain standards of acceptability in terms of

- significance of the topic
- definition of the problem
- design of the study
- control of variables
- sampling procedures
- use of instruments
- analysis of data
- interpretation of results
- reporting

This is not to say that criticism of them is unjustified. Each could be criticized on one or more points (in common with almost all research), and at some points in previous sections some of these criticisms were made. It has already been noted that lack of specificity about programs and procedures and lack of a control group characterize many studies. Length of time, number of students involved, and each of the other listed points denote problems. The greatest weakness of the research for our purposes, however, is that it was rarely focused specifically on urban children. Nevertheless, there is much evidence that relates to these children.

Summary

A. Introduction

- "The urban child" is a construct: The focus must be on individuals. (p. 4)
- It seems apparent that urban children do not learn mathematics differently from other children. (pp. 4-5)
- Most effective procedures are effective not only for urban children, but other children as well. (pp. 1-2)

B. The status of achievement of students in urban schools

- NAEP data indicate that children from the "low metropolitan" population scored significantly below the national average in mathematics, while those from the "high metropolitan" population scored consistently above the national average. (p. 9)
- Blacks scored far lower than whites on NAEP tests; the gap increased with age, according to an analysis of EOS data. Attainment for blacks appears to be deferred. (p. 12)
- Comparatively little research in mathematics education has focused directly on comparisons of urban and non-urban students--but many research findings pertain to them as well as to other students. (p. 12)

C. Evidence on existing practices in mathematic instruction

1. Organizational pattern

- No one pattern seems more effective than any other--belief (by teachers) that a pattern is effective and succeeding appears to be an essential factor. (p. 14)
- Good teachers can be effective regardless of the pattern. (p. 14)

2. Curriculum and content

- No indication was found that curriculum and content differ in major ways for urban children; minimal competencies are minimal competencies, although applications (taught, expected, and tested) should make use of the environment in which the child lives. (Selected projects are cited.) (pp. 15-21)

3. Class size

- Small classes may be particularly advantageous for non-white students. (p. 21)

4. Time allotment and use

- The amount of time actually spent on mathematics instruction may be only one-third of the time allocated for mathematics instruction (in the elementary school). (p. 22)
- Spending more than half the instructional time on developmental activities (in contrast to practice) probably results in higher achievement. (p. 23)

5. Teaching approaches

- It would appear that no one mode of instruction can be considered best; the teacher should learn as many as possible and use them to meet pupil needs. (p. 27)

- Meaningful instruction will lead to higher achievement, retention, and transfer than will rote instruction. (p. 27)
- Teachers believe that activities should be used, but few actually use them. (p. 27)
- Few variables consistently make a difference in school performance. (p. 28)

6. Differentiation of instruction

- Little evidence exists that self-paced programs are any more effective than other programs. (pp. 28-30)
- Many procedures for differentiating instruction have, however, been found to be effective--e.g., grouping to meet specific needs of individuals. (p. 30).
- Teachers frequently do not differentiate instruction, and may frequently select topics and ideas to teach which the students already know. (p. 30-31)

7. Student characteristics

a. Student aptitude

- Aptitude is highly correlated with mathematical achievement; specific mathematical abilities, language, sex, age, and heredity may also be important. (pp. 31-32)
- Individual differences in mathematical ability increase at successive age or grade levels. (p. 32)

b. Attitudes

- Children's attitudes toward mathematics appear to become increasingly less positive as they progress through school. (p. 35)

- While research indicates that positive or negative attitudes toward mathematics appear to have low correlations with achievement, mathematics educators and teachers believe that the affective component of learning is important. (p. 35)
- Teachers are widely believed to be prime determiners of a student's attitude and performance. (p. 35)

c. Self-concept

- Whether or not self-concept and achievement are related is unclear, but the development of a positive self-concept is important for all children. (p. 37)

c. Sex differences

- Few differences between boys' and girls' mathematical achievement are evident during the elementary-school years; no conclusion can be reached concerning high school learners, but certainly fewer girls continue to take mathematics courses in grades 11 and 12. (pp. 38-39)

e. Socioeconomic status

- Students from high socioeconomic levels tend to achieve better than students from low SES levels. (p. 39)
- Members of racial and ethnic minorities in general achieve as well or less well than members of the majority (white/Caucasian) group. (p. 39-40)
- SES is highly related to mathematical achievement, but may be less important for mathematics than for other curricular areas. (p. 40)

- A poor home environment combined with minority group status can lead to children who seem less capable of handling intellectual and linguistic tasks. (p. 42)
- Teachers of these children must emphasize:
 - The selection of initial learning materials geared to the learner's readiness level.
 - Mastery (consolidation) of tasks before new tasks are introduced.
 - Structural learning materials organized to facilitate efficient sequential learning. (p. 42)

f. The educationally disadvantaged student

- Preschool and primary compensatory education programs that carefully control the structure of the learning environment appear most successful. (p. 43)
- There is little evidence that compensatory education has been able to arrest the cumulative achievement deficit that exists between advantaged and disadvantaged students. (p. 47)
- The disadvantaged, as well as all other pupils, profit from special attention from the teacher, the content of the program, the instructional materials, and the organization for instruction. (p. 48)
- Mathematical characteristics which distinguish disadvantaged from advantaged pupils appear to exist in degree rather than kind; i.e., they have similar abilities and skills, but differ in depth or level of attainment. (p. 48)

g. The talented or gifted student

- Special courses and enrichment are the primary means of serving the interests of the exceptionally talented student in the secondary school, with acceleration next most often used. (p. 49)

- However, acceleration may be preferable to enrichment at the junior high level. (p. 49)

8. Instructional materials

a. Textbooks and other print materials

- The textbook is the primary determinant of mathematics curricula. (p. 50)
- Teachers follow the textbook closely with regard to content selection and sequencing, though they may skip components which they do not consider essential. They make little use of special features or topics not in the textbook. (p. 52)

b. Programmed instruction

- Programmed instruction is generally as effective as traditional instruction, but low achievers are often not sufficiently motivated to use programmed instruction independently. (p. 53)
- It is unclear whether pupils actually progress at individual rates with programmed instruction. (p. 53)

c. Manipulative materials

- Lessons using manipulative materials have a higher probability of producing greater mathematics achievement than do non-manipulative lessons. (p. 54)
- The use of materials appears to be as effective at one achievement level, or one ability level, as at another. (p. 55)
- Although the data are sparse, the use of materials appears to be as effective at one socioeconomic level as at another. (p. 55)

d. Computer-aided instruction

- At the elementary school level, computer-aided instruction is apparently effective as a supplement to regular instruction. At the secondary level it is about as effective as traditional instruction when it is used as a replacement. (p. 58)

e. Calculators

- Research to date indicates that achievement is higher or as high when calculators are used for instruction as when they are not used; children can learn mathematical ideas effectively with the calculators. (pp. 60-61)

9. Federal funding impact

- Federal monies have affected local school priorities, but it is difficult to ascertain their effect on learning and achievement. (pp. 61-62)

10. Progress assessments

- In most states having minimal competency testing, mathematics is one of the subjects tested; however, there is great variability in the mathematical objectives being assessed. (p. 65)
- A review of available assessment reports indicated that the topics with which difficulty were reported can be ranked in this order of frequency: fractions, division, subtraction with regrouping, decimals, geometry, measurement, proof estimation, statistics and probability, problem solving. This corresponds with information from previous error-analysis studies and studies on difficulties and the need for remediation. (p. 66)
- Status of students on mathematical assessments was reported as "at norm" and "below norm" in approximately an equivalent number of cases; fewer instances of "above norm" were noted. (p. 66)

- In other state, local, and regional assessments, some comparing data across a period of years, no clear trend can be observed. (p. 66)

D. Effective strategies for urban students

- Precise reasons for success with urban children could not always be discerned for such reasons as lack of specificity about the program and procedures and lack of a control group. (p. 68)
- Suggested guidelines for developing curricula include:
 - having urban school teachers on city-wide curriculum committees
 - designing curricula which provide for different student activities
 - designing curricula which provide for special needs of local-school students
 - emphasizing curriculum materials which require less reading
 - selecting cooperatively supplementary curriculum materials
 - placing experienced teachers of mathematics in urban schools
 - providing in-service sessions to develop understandings of urban children and to perfect mathematics instructional skills. (p. 69)
- Four key dimensions of mathematics programs, each significantly related to achievement gain, are: organizational climate, parent and community involvement, program characteristics, and resource use. (p. 69)
- Innovative school environments did not demonstrate a substantially positive impact on either achievement or attitudes. Educational quality is not synonymous with innovation or individualization. Students enrolled in programs with a more moderate emphasis on innovation and individualization showed greatest achievement gains. (p. 70)
- Associated with teacher effectiveness were: student-initiated behavior, whole-class instruction, general clarity of instruction, a non-evaluative and generally relaxed environment, higher achievement expectations, and classrooms that were relatively free

of major behavior disorders. Thus successful teaching appeared to be based on a large number of variables that must be present, to at least a minimal extent, rather than on one or two critical factors. (p. 70)

- Suggested components of programs for urban children include:
 - Teachers should have higher expectations for these children than they frequently do.
 - Students should be involved in learning.
 - Parents should be involved with their child's learning.
 - Natural language and language patterns should be used.
 - Increasing the physical comfort of learners is important.
 - Concepts should be "talked through" as an advance organizer before instruction per se.
 - Special times should be set aside for individual skill building.
 - Correction and checking of work should be immediate.
 - The student's self-image should be enhanced.
 - Careful planning should be done to provide "success experiences."
 - Diagnosis, prescription, and individualization are key components of programs.
 - Evaluation in terms of pre-determined goals should be an ongoing aspect of programs.
 - Teacher training must be directed toward work with urban students.
 - Programs must be relevant to the children. (p. 72)
- Differing values appear to lie at the base of the problem, and few teachers are trained to be aware of such differences and their implications for instruction. (p. 72)

Gaps in the evidence

What gaps exist in the research on the mathematical learning of urban children? It should be evident that the gaps are numerous--in common with the research on other children, we simply do not know or understand very much about:

- how children learn--and how they process mathematical concepts and ideas;
- what types of educational environment are likely to be most successful;
- what curricular and instructional modes are likely to be most successful;
- what teacher characteristics and preparation are likely to result in greatest success;
- what instructional aids are likely to be most successful;
- what needs are unique to urban children and how can they be met.

We know some practices which are effective, and some which are not. We need to search for the alternatives and for the combinations which will help more children to attain a better command of skills, concepts, and ideas in mathematics.

V. RESEARCH PRIORITIES

The gaps in the research evidence on mathematics education need to be filled, but it is obvious that this is not a short-term process. Priorities are needed; thus, the research on the mathematical needs of urban children and on procedures that work in meeting these needs, should be systematically planned. The mere infusion of dollars is not sufficient. Many aspects are not easily or immediately amenable to change--the amount of time for mathematics, teacher training, resources, the content fixed by texts and tests. But we must start.

- There should be an analysis of mathematics content, curriculum, and instructional practices in relation to the urban setting in which mathematics is taught.
- There should be an analysis of the real and perceived mathematical needs of students, in order to clarify concerns of relevancy, yet to assure that career options (which are broader when the amount of mathematics taken is greater) are not compromised.
- There should be in-depth analyses of teachers, classes, and schools in which successful programs and procedures are underway, in an attempt to ascertain the variables and components which will potentially affect their success elsewhere.
- There should be carefully defined studies of maximal importance in clarifying real issues, controlled not for the sake of experimental perfection but in order to be able to specify the parameters of success. It is particularly important to control the teacher variable--that is, to have a wide variety of teachers involved.

Thus, research on the curriculum and instruction in mathematics should focus on the learner under well defined conditions. The attributes of the learners should be specified, the specific characteristics of the mathematical learning task should be specified, and the procedures or activities which are designed to teach the task should be specified. Care should be taken not only to specify these facets, but also in many instances to have some means of comparison, some control group.

But before the comparative rigorousness of the experimental situation is reached, much exploration could be done, using the clinical-study mode. Case studies of individual children, classes, schools, teachers, programs, and communities could provide valuable clues for improving both cognitive and affective outcomes. Interviews with children to ascertain their understandings and their reactions to mathematics instruction on particular

portions of the curriculum add another factor to our understanding. The clinical investigations can be used to generate hypotheses for rigorous testing. Without such hypotheses, rigorous experimentation would be premature in contributing specific, replicable information.

There are two pressing concerns in relation to mathematics education: (1) what should be taught and (2) how it should be taught. The mathematics curriculum is (once again) under attack by those who support a move "back to the basics." What they mean is somewhat nebulous, but most seem to agree that computational skills are the basics. Computational skills are important; computational skills will continue to be taught--there is no question about that. But to limit the elementary school curriculum to computational skills ignores the needs of children in an age which has become increasingly technological. We must have a mathematically literate population, and this means more than competency with computational skills. Mathematical literacy encompasses at least the 10 areas listed by the National Council of Supervisors of Mathematics (1977):

- problem solving
- applying mathematics to everyday situations
- alertness to the reasonableness of results
- estimation and approximation
- appropriate computational skills
- geometry
- measurement
- reading, interpreting, and constructing tables, charts, and graphs
- using mathematics to predict
- computer literacy

Research can probably do little to convince people of the need for a curriculum which includes more than computational skills. But we can apply some of the research evidence, and the evidence from successful practice, which will assure that children successfully achieve computational skills.

A number of references are available which present this research evidence--e.g., Classroom Ideas from Research on Computational Skills (Suydam & Dessart, 1976); the Thirty-seventh NCTM Yearbook, Mathematics Learning for Early Childhood (Payne, 1975); and the 1978 NCTM Yearbook, Developing Computational Skills (Suydam, 1978). In the latter, for instance, is an article listing guidelines for drill work.

Drill and practice are necessary components of mathematics instruction, but not the only ones. The second pressing concern is related directly to them: how mathematics should be taught. Some persons have advocated a return to a drill method of instruction, with rote teaching--that is, telling or showing the child how to perform a task and then having the child practice until the task is mastered, without any explanation for why the procedure works or is true--and an atomized curriculum, one in which content is divided into small bits and fed into the child. Such persons do not seem aware that just such a method was used through the 1930s--and was the source of widespread dissatisfaction. But the evidence that supported a change to more meaningful methods of instruction needs to be replicated in terms of today's children and today's curriculum. And the question of concern should be, What is the (optimal) blend of rote and meaningful methods? This acknowledges that perhaps some rote instruction has a place in mathematics instruction. But it also acknowledges that meaningful instruction is necessary, particularly as long-range goals are considered. "Direct instruction" is a form of rote instruction; it has an immediate payoff in terms of achievement. The questions that need to be asked are, How does such instruction affect the long-range goals of (1) retention and transfer and (2) mathematical literacy?

It seems evident that the mathematics program is not the primary source of concern in urban schools. Discipline, control, and management and, related in both subtle and overt ways, student reactions, attitudes, and motivation

are of vital importance. If the teacher cannot capture the child's attention, or if the student is not even in school, there is little hope that the child will learn what is being taught. Since this paper reviewed the research specific to mathematics education, the discipline factor rarely surfaced. Needless to say, however, it has been the concern of mathematics teachers and other educators.

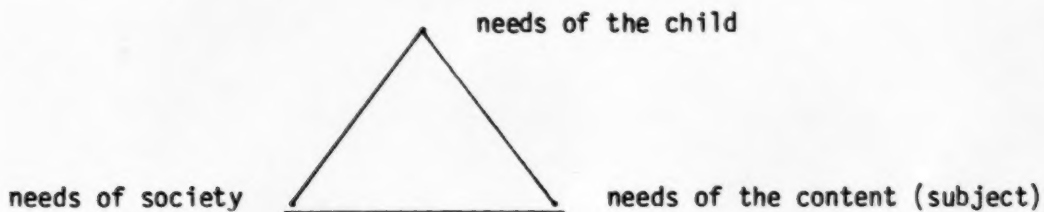
The importance of teacher involvement cannot be taken too lightly. The teacher must be convinced that students can learn. A study of a first-grade teacher by Pedersen et al. (1978), while it could be criticized on several research grounds, nevertheless provides some evidence on the effect of a single teacher. At each succeeding level and on to the vocational level, the influence of one first-grade teacher on the achievement of her pupils was evident. Affective factors seem to account for much of the difference, although this, unfortunately, is not documented in the study (it is difficult to document the unrecorded past of everyday experience). The teacher has standards; the teacher expected children, all children, to learn; the teacher let each child know how well he or she was doing. Praise was one of the components that seemed particularly important in retrospect. The self-image or self-concept of the child and the manner and the attitude of the teacher are vital ingredients of learning.

Perhaps the magic word is "rapport," noted by outside evaluators of Title I programs as a component of successful programs. Related to it in some instances is the concept of "the whole child," a philosophical outlook that encompasses the notion that we need to learn about the child from various facets--psychological, emotional, social, as well as intellectual. Teachers could profit from more emphasis on the first three of these in their training programs.

In summary, there are two important and pressing questions which need to be addressed:

1. What is the scope of what we should teach?
2. What teaching methods should be used?

Each reflects positions of disagreement: back-to-the-basics versus mathematical literacy, rote versus meaningful instruction. The split is significant, particularly because professionals and the lay public (i.e., taxpayers) reflect opposing points of view about each concern. In addition, there is intra-professional disagreement on the importance of the needs of the child versus the needs of the content. Thus the triangle proposed perhaps by Dewey, and later by Glennon, is being tugged from each direction:



The hypotheses that need to be examined include:

1. A broad interpretation of mathematical literacy is needed.
2. A blend of rote and meaningful methods of instruction is needed.

They should be examined first by clinical approaches, including case studies and interviews to obtain in-depth information. Then appropriately controlled experimentation should be conducted. The most important thing in each is to record what occurs so that replicability with other children, teachers, and schools will be possible.

VI. RECOMMENDATIONS

Three general recommendations bear restating:

- The curriculum for "everychild" should be aimed at mathematical literacy rather than only minimal computational skills. The needs of individuals should be considered, not merely the needs of the "average" child.

- Perhaps we have "averaged out" the effects of particular instructional procedures and curricula for urban children: Averages may have disguised their true impact. We need data on the experiences of specific teachers confronting specific pupils, not data about the average level of child or teacher. Research on mathematics for urban children should be focused on mathematics in the urban setting.
- The advances of technology, in particular the calculator and the computer, open new possibilities for every person, and these must be considered as curricula are reexamined.

When the automobile came along, we thought it would just get around a little more efficiently than the horse. It did that, but it also changed lives completely. It changed the way we work, play, and build cities. It changed our lives in fundamental ways we could not predict at the time. I think the same thing will happen with the personal computer. The real impact, I believe, will be in ways we can't even guess now. When everyone has what amounts to unlimited computing power, human ingenuity will find hundreds, thousands, millions of ways to use it. (Gilmore, 1978)

The NCTM statement included as the preface to this paper indicated directions that must be taken in terms of pre- and inservice education to aid the teacher in the urban school. They are repeated here for emphasis; urban teachers should have:

- low teacher-pupil ratio so that individual attention is possible;
- an unbiased testing program;
- materials appropriate to the age and ability of students;
- a supportive relationship between community and school;
- ease of introduction for beginning teachers;
- training that provides:
 - methods for teaching students with a wide range of ability

- methods for implementing activity-oriented programs;
- methods for reaching the unmotivated, for enhancing students' self-image, for raising academic aspirations and expectations of students;
- classroom-management techniques;
- information about special characteristics of urban students;
- experience in the social milieu of the community;
- special language instruction when appropriate;
- means to work within the system to the advantage of students;
- reasonable self-evaluation techniques.

Lack of understanding by the teacher of different ethical patterns and beliefs, not only of racial or ethnic groups, but of children from a different class from their own, is widespread. Taking into consideration the values of the students, their needs for mathematics, and their aspirations for the future, and helping them to understand the relative importance of mathematical ideas are essential.

REFERENCES

- Abt Associates. Education as experimentation: A planned variation model (Volume III). Cambridge, Massachusetts: Abt Associates, 1976.
- Abt Associates. Education as experimentation: A planned variation model (Volume IV). Cambridge, Massachusetts: Abt Associates, 1977.
- Aiken, L. R., Jr. Attitudes toward mathematics. Review of Educational Research, October 1970, 40: 551-596.
- Aiken, L. R., Jr. Intellective variables and mathematics achievement: Directions for research. Journal of School Psychology 1971, 9, 201-202.
- Aiken, L. R., Jr. Research on attitudes toward mathematics. Arithmetic Teacher, March 1972, 19, 229-234.
- Albers, D. F. An investigation of the effects of the allocation of class time on pupil achievement and scheduling preferences. (University of Missouri-Columbia, 1972.) Dissertation Abstracts International, 1973 33A, 4700.
- Alspaugh, J. W. A survey of secondary mathematic programs in Missouri with emphasis on content, procedures, and preparation of teachers. (University of Missouri, 1965.) Dissertation Abstracts, 1966, 26, 5259-5260.
- Anttonen, R. G. An examination into the stability of mathematics attitude and its relationship to mathematics achievement from elementary to secondary school level. (University of Minnesota, 1967.) Dissertation Abstracts, 1968, 28A, 3011-3012.
- Asbury, C. A. Factors associated with discrepant achievement in rural economically deprived white and negro first graders. (University of North Carolina at Chapel Hill, 1969.) Dissertation Abstracts, 1970, 31A, 208-209.
- Austin, G. R., Rogers, B. G., & Walbesser, H. H., Jr. The effectiveness of summer compensatory education: A review of the research. Review of Educational Research, 1972, 42, 171-181.
- Ausubel, D. P. How reversible are the cognitive and motivational effects of cultural deprivation? Implications for teaching the culturally deprived child. Urban Education, 1964, 1, 16-38.

- Bachman, A. M. Factors related to the achievement of junior high school students in mathematics. (University of Oregon, 1968.) Dissertation Abstracts, 1969, 29A, 2139.
- Barley, S. D. Time: In school, and learning. (The University of Rochester, 1975.) Dissertation Abstracts International, 1975, 36A, 2583-2584.
- Becker, W. C. Teaching reading and language to the disadvantaged--What we have learned from field research. Harvard Educational Review, 1977, 47, 518-543.
- Begle, E. G. Learning correlates for Mexican-American students. SMESG Working Paper No. 12. Stanford, California: Stanford Mathematics Education Study Group, November 1972. (ERIC: ED 142 406)
- Begle, E. G. Some lessons learned by SMSG. Mathematics Teacher, 1973, 66, 207-214.
- Begle, E. G. Acceleration for students talented in mathematics. Working Paper No. 19. Stanford, California: Stanford University, Mathematics Education Study Group, February 1976. (ERIC: ED 121 607)
- Begle, E. G., Pence, B. J. et al. Test factors, instructional programs, and socio-cultural-economic factors related to mathematics achievement of Chicano students: A review of the literature. SMESG Working Paper No. 10. Stanford, California: California School Mathematics Education Study Group, March 1975. (ERIC: ED 113 198)
- Birr, D. J. The effects of the treatments by parents and teachers on the self-concept of ability held by underachieving early adolescent pupils. (Michigan State University, 1969.) Dissertation Abstracts International, 1969, 30A, 1354.
- Bishop, T. D. A study of the computer-related mathematics programs of secondary schools and teacher education institutions in Missouri and adjoining states. (University of Missouri-Columbia, 1970.) Dissertation Abstracts International, 1971, 31A, 3997-3998.
- Boardman, A. E. et al. A simultaneous equations model of the educational process: The Coleman data revisited with an emphasis on achievement. Washington: American Statistical Association, 1973. (ERIC: ED 097 404)
- Bobier, D. T. The effectiveness of the independent use of programmed textbooks in aiding students to overcome skill weaknesses in English mechanics and arithmetic. (University of Denver, 1964.) Dissertation Abstracts, 1964, 25, 3424-3425.

- Bogatz, G. A., & Ball, S. A summary of the major findings in "The second year of Sesame Street: A continuing evaluation." New York: Children's Television Workshop, and Princeton: Educational Testing Service, 1971. (ERIC: ED 122 802)
- Bredemeier, H. C. The differential effectiveness of high schools with selected characteristics in producing cognitive growth in different kinds of students. New Brunswick, New Jersey: Rutgers, The State University, June 1967. (ERIC: ED 014 771)
- Bright, G. W. Assessing the development of computation skills. In M. N. Suydam (Ed.), Developing computational skills (1978 NCTM Yearbook). Reston, Virginia: National Council of Teachers of Mathematics, 1978.
- Brown, J. K., Jr. Textbook use by teachers and students of geometry and second-year algebra. (University of Illinois at Urbana-Champaign, 1973.) Dissertation Abstracts International, 1974, 34A, 5795-5796.
- Buchalter, B. D. The validity of mathematics textbook series in grades 7-14 with structure as an objective. (University of Arizona, 1968.) Dissertation Abstracts International, 1969, 30A, 198-199.
- Buchman, A. L. Some relationships between length of courses in elementary algebra and student characteristics. (State University of New York at Albany, 1972.) Dissertation Abstracts International, 1972, 33A, 2812.
- Bukoski, W. J., & Korotkin, A. L. Computing activities in secondary education. Educational Technology, 1976, 16, 9-23. (See also ERIC: ED 112 935)
- Burns, P. C. Arithmetic books for elementary schools. Arithmetic Teacher, 1960, 7, 147-149.
- Callahan, L. G., & Glennon, V. J. Elementary school mathematics, a guide to current research. 4th. ed. Washington: Association for Supervision and Curriculum Development, 1975.
- Cambridge Conference. Goals for school mathematics, the report of the conference on school mathematics (Cambridge, Massachusetts, 1963). Watertown, Massachusetts: Educational Services, Inc., 1963. (ERIC: ED 015 140)
- Carpenter, T. P., Coburn, T. G., Reys, R. E., & Wilson, J. W. Notes from National Assessment. Arithmetic Teacher, 1975-1976, 22, 23.
- Castaneda, A. M. The differential effectiveness of two first grade mathematics programs for disadvantaged Mexican-American children. (The University of Texas, 1967.) Dissertation Abstracts, 1968, 28A, 3878-3879.

- Centrone, J. J., Sr. Teacher sociocultural awareness in selected schools in New York State accountable for American Indian education. (Syracuse University, 1972.) Dissertation Abstracts International, 1973, 33A, 5596-5597.
- Chalupsky, A. B., & Coles, G. J. Exploring the impact of educational innovation: Overview of Project LONGSTEP. Palo Alto, California: American Institutes for Research in the Behavioral Sciences, 1976. (ERIC: ED 132 191)
- Chalupsky, A. B. & Coles G. J. The unfulfilled promise of educational innovation. Paper presented at the annual meeting of the American Educational Research Association, New York, April 1977. (ERIC: ED 137 942)
- Clason, R. G. Number concepts in arithmetic texts of the United States from 1880 to 1966, with related psychological and mathematical developments. (The University of Michigan, 1969.) Dissertation Abstracts International, 1969, 30A, 146.
- Coleman, J. S. et al. Equality of educational opportunity. Washington, D.C.: U.S. Government Printing Office, 1966.
- Coles, G. J., & Chalupsky, A. B. Impact of educational innovation on student performance: Overall findings for reading and arithmetic (Project LONGSTEP Final Report: Vol. I, supplement). Palo Alto, California: American Institutes for Research in the Behavioral Sciences, September 1976. (ERIC: ED 132 192)
- Coles, G. J. et al. Impact of educational innovation on student performance: Project methods and findings for three cohorts (Project LONGSTEP Final Report: Vol. I). Palo Alto, California: American Institutes for Research in Behavioral Sciences, April 1976. (ERIC: ED 132 177)
- Conant, E. H. Teacher and paraprofessional work productivity, a public school cost-effectiveness study. Lexington, Massachusetts: D. C. Heath, 1973.
- Crosswhite, F. J. Correlates of attitudes toward mathematics. (NLSMA Report No. 20.) Stanford, California: School Mathematics Education Study Group, 1972.
- Crosswhite, F. J. (Ed.). Organizing for mathematics instruction. (1977 Yearbook). Reston, Virginia: National Council for Teachers of Mathematics, 1977.

- Dahle, M. M. A procedure for the measurement of the content validity of standardized tests in elementary mathematics. (University of Southern California, 1970.) Dissertation Abstracts International, 1970, 30A, 5336.
- Darby, C. A., Korotkin, A. L., & Romashko, T. Survey of computing activities in secondary schools. Washington: American Institutes for Research, R70-13, 1970. (See also ERIC: ED 047 500, ED 069 540)
- DeShields, J. I. Factors effecting achievement in ESEA Title I schools and non-ESEA Title I schools. (University of Massachusetts, 1973.) Dissertation Abstracts International, 1973, 33A, 6585.
- Deutsch, M. The role of social class in language development and cognition. American Journal of Orthopsychiatry, 1965, 35, 78-87.
- Dunkley, M. E. Some number concepts of disadvantaged children. Arithmetic Teacher, May 1965, 12, 359-361.
- Edmonds, R. R. et al. Search for effective schools; the identification and analysis of city schools that are instructionally effective for poor children, 1977. (ERIC: ED 142 610)
- Ellison, R. L. et al. Utah statewide educational assessment: General report. Salt Lake City: Utah State Board of Education, December 1975.
- EPIE. Research findings: More about NSAIM. EPIEgram 5 (2), October 15, 1976 (a).
- EPIE. Research findings: NSAIM: Two years later. EPIEgram 5 (5), December 1, 1976 (b).
- EPIE. Doubts about individualized instruction. EPIEgram 5 (10), February 15, 1977.
- Ernest, J. Mathematics and sex. American Mathematical Monthly, 1976, 83, 595-614.
- Ernest, J. et al. Mathematics and sex. Santa Barbara: University of California at Santa Barbara, January 1975. (ERIC: ED 107 535)
- Evans, E. D. Contemporary influences in early childhood education. New York: Holt, Rinehart and Winston, 1971.
- Everett, B. E. A preliminary study of the relevance of a standardized test for measuring achievement gains to innovative arithmetic programs. (Project LONGSTEP Final Report. Volume II. Appendix Report). Palo Alto, California: American Institutes for Research in the Behavioral Sciences, September 1976. (ERIC: ED 132 180)

- Feierabend, R. L. Review of research on psychological problems in mathematics education. In Research problems in mathematics education. Cooperative Research Monograph No. 3, U.S. Office of Education, 1960.
- Fennema, E. Mathematics learning and the sexes: A review. Journal for Research in Mathematics Education, 1974, 5, 126-139.
- Fennema, E. & Sherman, J. Sex-related differences in mathematics learning: Myths, realities and related factors. February 1976. (ERIC: ED 129 633)
- Fey, J. T. Patterns of verbal communication in mathematics classes. (Columbia University, 1968.) Dissertation Abstracts International, 1969, 29A, 3040.
- Filby, N. N., Marliave, R., & Fisher, C. W. Allocated and engaged time in different content areas of second and fifth grade reading and mathematics curriculum, 1976. (ERIC: ED 137 315)
- Fisher, C. W., Filby, N. N., & Marliave, R. Instructional time and student achievement in second grade reading and mathematics, 1976 (a). (ERIC: ED 137 293)
- Fisher, C. W. et al. A study of instructional time in grade 2 mathematics. San Francisco, California: Far West Laboratory for Educational Research and Development, 1976 (b).
- Folsom, M. Teachers look at arithmetic manuals. Arithmetic Teacher, 1960, 7, 13-18.
- Fox, L. H. Facilitating the development of mathematical talent in young women. (The Johns Hopkins University, 1974.) Dissertation Abstracts International, 1975, 35B, 3553.
- Freda, J. A. Measures of socioeconomic complexities of California school communities as related to student achievement. (United States International University, 1976.) Dissertation Abstracts International, 1976, 37A, 103.
- Furno, O. F., & Collins, G. J. Class size and pupil learning. Baltimore, Maryland: Baltimore City Public Schools, 1967.
- Gates, J. O., Jr. A survey of time allotment practices in the elementary school as reported by classroom teachers in thirty public school systems in Oklahoma. (The University of Oklahoma, 1968.) Dissertation Abstracts, 1969, 29A, 2604.
- Gilmore, C. P. PS: What's news. Popular Science, March 1978, 212, 65.

- Good, T. L., Biddle, B. J., & Brophy, J. E. Teachers make a difference. New York: Holt, Rinehart and Winston, 1975.
- Good, T. L., & Grouws, D. A. Process-product relationships in fourth grade mathematics classrooms. Washington, D.C.: National Institute of Education (DHEW), October 1975.
- Good, T. L., & Grouws, D. A. Teaching effects: A process-product study in fourth-grade mathematics classrooms. Journal of Teacher Education, May-June 1977, 28, 49-54.
- Goodlad, J. I. What goes on in our schools? Educational Researcher, March 1977, 6, 3-6.
- Goodlad, J. I., Klein, M. F., & Associates. Behind the classroom door. Worthington, Ohio: Charles A. Jones, 1970.
- Graeber, A. O., Rim, E., & Unks, N. J. A survey of classroom practices in mathematics: Reports of first, third, fifth and seventh grade teachers in Delaware, New Jersey, and Pennsylvania. Philadelphia: Research for Better Schools, Inc., June 1977.
- Green, R. Math avoidance: A barrier to American Indian science education and science careers. In Report and recommendations: Conference on mathematics in American Indian Education. Washington: American Association for the Advancement of Science, February 1978.
- Green, R. W. A survey of the mathematical instructional materials used in teaching culturally disadvantaged children in grades 1 through 6 throughout the United States. (Indiana University, 1969.) Dissertation Abstracts International, 1970, 31A, 1101.
- Haladyna, T. Statewide survey of the impact of OSME. Monmouth, Oregon: Teaching Research, November 1975.
- Halperin, M. S. First-grade teachers' goals and children's developing perceptions of school. Journal of Educational Psychology, 1976, 68, 636-648.
- Hansen, V. P. Elementary algebra achievement as related to class length and teaching method. (University of Minnesota, 1962.) Dissertation Abstracts, 1963, 24, 198.
- Harnischfeger, A., & Wiley, D. E. Achievement test score decline: Do we need to worry? Chicago: CEMREL, Inc., 1975.
- Hartley, S. S. Meta-analysis of the effects of individually paced instruction in mathematics. (University of Colorado at Boulder, 1977.) Dissertation Abstracts International, 1978, 38A, 4003-4004.

- Hatfield, L. L. Computer-extended problem solving and enquiry. Columbus, Ohio: ERIC Information Analysis Center for Science, Mathematics, and Environmental Education, February 1973. (ERIC: ED 077 732)
- Hawthorne, P. Legislation by the states: Accountability and assessment in education. Revised (Report No. 2. Bulletin No. 3100). Denver: Colorado State Department of Education; Cooperative Accountability Project and Madison: Wisconsin State Department of Public Instruction, Division of Management Planning Services, November 1974. (ERIC: ED 098 681)
- Hayes, E. J. Relationships between self-concept of arithmetic ability and arithmetic achievement in a selected group of sixth grade students. (Michigan State University, 1967.) Dissertation Abstracts, 1968, 28A, 3999.
- Herriot, S. F. The secondary school "slow-learner" in mathematics. (Stanford University, 1967.) Dissertation Abstracts, 1968, 28A, 3072-3073.
- Hess, R. D. & Tenezakis, M. D. Selected findings from "The computer as a socializing agent: Some socioaffective outcomes of DAI." AV Communication Review, 1973, 3, 311-325.
- Hughes, M. M. et al. Assessment of the quality of teaching in elementary schools. Salt Lake City: University of Utah, 1959.
- Hunter, M. L. Group effect on self-concept and math performance. (California School of Professional Psychology, Los Angeles, 1973.) Dissertation Abstracts International, 1974, 34B, 5169.
- Hushak, L. J. The contribution of school and non-school inputs to student achievement. (Final Report) Columbus: Ohio State University Research Foundation, September 1973. (ERIC: ED 085 410)
- Hushak, L. J. The role of schools in reducing the variance of cognitive skills. Journal of Educational Research, 1977, 70, 115-122.
- Jamison, D., Suppes, P., & Wells, S. The effectiveness of alternative instructional media: A survey. Review of Educational Research, 1974, 44, 1-67.
- Jarvis, O. T. Time allotment relationships to pupil achievement in arithmetic. Arithmetic Teacher, 1963, 10, 248-250.
- Jarvis, O. T. Arithmetic and science time allotment practices in intermediate grades. School Science and Mathematics, 1966, 66, 322-324.

- Jencks, C. S. & Brown, M. D. Effects of high schools on their students. Harvard Educational Review, 1975, 45, 273-324.
- Johnson, R. T., Jr. A comparison of categorizing ability in high and low socioeconomic kindergarteners. (University of California, Berkeley, 1969.) Dissertation Abstracts International, 1970, 31A, 225.
- Johnston, A. M. A survey of teaching practices--arithmetic in Tennessee in the first through eighth grades. Arithmetic Teacher, 1962, 9, 425-432.
- Kahn, H. F. A study of the manner in which selected topics in elementary algebra were presented to students in America between 1900 and 1970 as revealed in selected commercially published textbooks. (Temple University, 1974.) Dissertation Abstracts International, 1974, 35B, 1320-1321.
- Karweit, N. Quantity of schooling: A major education factor? Educational Researcher, February 1976, 5, 15-17.
- Kennedy, M. M. Findings from the Follow Through planned variation study. Educational Researcher, June 1978, 7, 3-11.
- Kester, S. W. The communication of teacher expectations and their effects on the achievement and attitudes of secondary school pupils. (The University of Oklahoma, 1969.) Dissertation Abstracts International, 1969, 30A, 1434-1435.
- Kieren, T. E. Activity learning. Review of Educational Research, 1969, 39, 509-522.
- Kieren, T. E. Manipulative activity in mathematic learning. Journal for Research in Mathematics Education, 1971, 2, 228-234.
- Kieren, T. E. Research on computers in mathematics education. Columbus, Ohio: ERIC Information Analysis Center for Science, Mathematics, and Environmental Education, April 1973. (ERIC: ED 077 734)
- Knaupp, J. Are children's attitudes toward learning arithmetic really important? School Science and Mathematics, 1973, 73, 9-15.
- Koch, D. R. Concept of self and mathematics achievement. (Auburn University, 1972.) Dissertation Abstracts International, 1972, 33A, 1081.
- Kozak, M. R. A critical analysis of individualized instruction since 1944. (Texas A&M University, 1974.) Dissertation Abstracts International, 1975, 35A, 2518-2519.

- Lachat, M. A., Capasso, R. L., & Bartinique, I. S. Math programs that work: A national survey. Camden, New Jersey: Capla Associates, Inc., 1977.
- Lackner, L. M. Teaching machines and programmed instruction. AV Communications Review, 1967, 115, 181-198.
- Lawson, J. B. Achievement differences in fourth grade under two time allotments and two sequences for introducing multiplication facts. (University of California, Berkeley, 1966.) Dissertation Abstracts, 1966, 27A, 995-996.
- Leiderman, G. F., Chinn, W. G., & Dunkley, M. E. The special curriculum project: Pilot program on mathematics learning of culturally disadvantaged primary school children. SMSG Reports, No. 2. Stanford, California: School Mathematics Study Group, 1966.
- Lowry, W. C. The slow learner in mathematics. (Thirty-fifth Yearbook) Washington: National Council of Teachers of Mathematics, 1972.
- Mahan, E. A. A study of the verbal behavior of four student teachers teaching selected geometric concepts to kindergarten children. (Columbia University, 1970.) Dissertation Abstracts International, 1971, 31A, 3392-3393.
- Manzo, A. V. The math student/the math teacher/the math problem, 1975. (ERIC: ED 114 767)
- Marcus, A. C. et al. Administrative leadership in a sample of successful schools from the national evaluation of the Emergency School Aid Act. Santa Monica, California: System Development Corporation, April 1976. (ERIC: ED 125 123)
- Marliave, R., Fisher, C. W., & Filby, N. N. Alternative procedures for collecting instructional time data: When can you ask the teacher and when must you observe for yourself? 1976. (ERIC: ED 137 380)
- Maura, C. A survey of the presentation of certain topics in ten series of arithmetic textbooks. (University of Maryland, 1957.) Dissertation Abstracts, 1957, 17, 1515-1516.
- McDaniel, R. The identification and description of changes in mathematics programs in the secondary schools in Tennessee which have used NDEA Title III funds. (The University of Tennessee, 1972.) Dissertation Abstracts International, 1973, 33B, 5395-5396.
- McLaughlin, M. W. Evaluation and reform: The Elementary and Secondary Education Act of 1965, Title I. Cambridge, Massachusetts: Ballinger, 1975.

- McLaughlin, M. W. Implementation as mutual adaptations: Change in classroom organization. Teachers College Record, 1976, 77, 339-351.
- McWhorter, E. W. The small electronic calculator. Scientific American, March, 1976, 234, 88-89.
- Meckes, R. C. A study to ascertain the instructional index and questioning strategy of mathematic teachers in grade 6, and to determine their relationship to professional characteristics and situational factors. (Southern Illinois University, 1971.) Dissertation Abstracts International, 1972, 32A, 4245-4246.
- Messer, S. B. The relation of internal-external control to academic performance. Child Development, 1972, 43, 1456-1462.
- Miller, G. H. How much time for arithmetic? Arithmetic Teacher, 1958, 5, 256-259.
- Miller, R. L. Individualized instruction in mathematics: A review of research. Mathematics Teacher, 1976, 69, 345-351.
- Montague, D. O. Arithmetic concepts of kindergarten children in contrasting socio-economic areas. Elementary School Journal, 1964, 64, 393-397.
- Moody, W. B., Bausell, R. B. & Jenkins, J. R. The effect of class size on the learning of mathematics: A parametric study with fourth-grade students. Journal for Research in Mathematics Education, 1973, 4, 170-176.
- Moore, B. D. The relationship of fifth-grade students' self-concepts and attitudes toward mathematics to academic achievement in arithmetical computation, concepts, and application. (North Texas State University, 1971.) Dissertation Abstracts International, 1972, 32, 4425.
- Moran, T. P. Significant developments in the use of computers in school mathematics: A sourcebook for administrators, teachers, and teacher educators. (Columbia University, 1974.) Dissertation Abstracts International, 1974, 35A, 2104.
- NACOME. Overview and analysis of school mathematics, grades K-12. Washington: National Advisory Committee on Mathematical Education, Conference Board of the Mathematical Sciences, 1975.
- NAEP. Math fundamentals: Selected results from the First National Assessment of Mathematics. Denver: National Assessment of Educational Progress, Education Commission of the States, January 1975 (a). (ERIC: ED 111 696)

- NAEP. Consumer math: Selected results from the First National Assessment of Mathematics. Denver: National Assessment of Educational Progress, Education Commission of the States, January 1975 (b). (ERIC: ED 111 696)
- NAEP. National Assessment of Educational Progress. The First National Assessment of Mathematics: An overview. Washington: National Center for Education Statistics, October 1975 (c). (ERIC: ED 127 198)
- NAEP. Mathematic technical report: Summary volume. Denver: National Assessment of Educational Progress, Education Commission of the States, September 1976. (ERIC: ED 129 636)
- NAEP. Survey views tv, homework habits. NAEP Newsletter, April 1978, 11, 1,3.
- NCSM. Position paper on basic mathematical skills. National Council of Supervisors of Mathematics, 1977. Reprinted in Arithmetic Teacher, October 1977, 25, 18-22, and Mathematics Teacher, 1978, 71, 147-152.
- NCTM. Position statement on minicalculators. Reston, Virginia: National Council of Teachers of Mathematics, 1974.
- Neale, D. C. The role of attitudes in learning mathematics. Arithmetic Teacher, December 1969, 16, 631-640.
- Neatrou, C. R. Geometric content in the mathematics curriculum of the middle school. (Indiana University, 1968.) Dissertation Abstracts, 1969, 29A, 3531-3532.
- Nelson, T. S. Factors present in effective teaching of secondary school mathematics. (University of Nebraska Teachers College, 1959.) Dissertation Abstracts, 1960, 20, 3207-3208.
- NIE. The NIE conference on basic mathematical skills and learning (Volume I: Contributed Position Papers; Volume II: Reports from the Working Groups). Washington: National Institute of Education, 1975. (ERIC: ED 125 908, ED 125 909)
- NIE/NSF. Report of the conference on needed research and development on hand-held calculators in school mathematics. Washington: National Institute of Education and National Science Foundation, 1977. (ERIC: ED 139 665)
- Norris, E. L., & Bowes, J. E. (Eds.). National Assessment of Educational Progress, mathematics objectives. Ann Arbor, Michigan: National Assessment of Educational Progress, 1970. (ERIC: ED 063 140)

- Okada, T. et al. Growth in achievement for different racial, regional and socio-economic groupings of students. Washington, D.C.: Office of Education (DHEW), Office of Program Planning and Evaluation, 1969. (ERIC: ED 032 211)
- Olson, M. N. Identifying predictors of quality: An examination of eleven internal classroom variables in relation to a school system criterion measure. (Columbia University, 1970.) Dissertation Abstracts International, 1971, 31A, 3226.
- Ordonez, J. M. School-related attitudes of Title I children in the District of Columbia public schools. (The George Washington University, 1971.) Dissertation Abstracts International, 1971, 32A, 1196.
- Osborne, A. & Nibbelink, W. Directions of curricular change. In J. N. Payne (Ed.), Mathematics learning in early childhood, (Thirty-seventh Yearbook). Reston, Virginia: National Council of Teachers of Mathematics, 1975, pp. 274-294.
- Passy, R. A. Socio-economic status and mathematics achievement. Arithmetic Teacher, 1964, 11, 469-470.
- Payne, J. N. (Ed.). Mathematics learning in early childhood (Thirty-seventh Yearbook). Reston, Virginia: National Council of Teachers of Mathematics, 1975.
- Pedersen, E., Faucher, T. A., & Eaton, W. W. A new perspective on the effects of first-grade teachers on children's subsequent adult status. Harvard Educational Review, February 1978, 48, 1-31.
- Phelan, E. J. Achievement, self-concept, creativity and attitude toward school of students in formal and informal education programs. (Fordham University, 1974.) Dissertation Abstracts International, 1974, 35A, 1400-1401.
- Pigge, F. L. An experimental comparison of three methods of teaching addition and subtraction of fractions in grade five. (Ohio University, 1964.) Dissertation Abstracts, 1964, 25, 1789-1790.
- Pipho, C. Update V: Minimal competency testing. Denver: Education Commission of States, April 1977.
- Poffenberger, T. & Norton, D. A. Factors in the formation of attitudes toward mathematics. Journal of Educational Research, 1959, 52, 171-176.
- Posamentier, A. S. Mathematical achievement and attitudinal differences among students and attitudinal differences among teachers under a two-semester and a three-semester elementary algebra course. (Fordham University, 1973.) Dissertation Abstracts International, 1973, 34A, 2279-2280.

- Primavera, L. H., Simon, W. E., & Primavera, A. M. The relationship between self-esteem and academic achievement: An investigation of sex differences. Psychology in the Schools, 1974, 11, 213-216.
- Price, J., Kelley, J., & Kelley, J. L. Description of Survey. In Overview and analysis of school mathematics grades K-12 (NACOME report). Washington: Conference Board of the Mathematical Sciences, National Advisory Committee on Mathematical Education, 1975, pp. 68ff.
- Rasmussen, D. S. Urban junior high school mathematics curricula at the seventh and eighth grade levels. Los Angeles: University of Southern California, Los Angeles, 1968. (ERIC: ED 028 932)
- Robertson, H. C. The effects of the discovery and expository approach of presenting and teaching selected mathematical principles and relationships to fourth grade pupils. (University of Pittsburgh, 1970.) Dissertation Abstracts International, 1971, 31A, 5278-5279.
- Roderick, S. A. A comparative study of mathematics achievement by sixth graders and eighth graders, 1936 to 1973, 1951-55 to 1973, and 1965 to 1973. (The University of Iowa, 1973.) Dissertation Abstracts International, 1974, 34A, 5601-5602.
- Rosenbloom, P. C., Torrance, E. P., & Flanders, N. A. Characteristics of mathematics teachers that affect students' learning. Minneapolis: University of Minnesota, September 1966. (ERIC: ED 021 707)
- Rudolph, E. L. A survey of data processing and computer use in instruction in Illinois secondary schools. (Northern Illinois University, 1972.) Dissertation Abstracts International, 1972, 33A, 505-506.
- Salopek, T. F. A study of academic achievement and school district characteristics. (State University of New York at Buffalo, 1973.) Dissertation Abstracts International, 1974, 34A, 6328-6329.
- Schoen, H. L. Self-paced mathematics instruction: How effective has it been? Arithmetic Teacher, 1976, 23, 90-96 (a).
- Schoen, H. L. Self-paced mathematics instruction: How effective has it been in secondary and postsecondary schools? Mathematics Teacher, May 1976, 69, 352-357 (b).
- Schoen, H. L. Implications of research for instruction in self-paced mathematics classrooms. In F. J. Crosswhite, (Ed.), Organizing for mathematics instruction (1977 Yearbook). Reston, Virginia: National Council of Teachers of Mathematics, 1977. pp. 198-223.
- Shetler, L. L. Practices and trends in the teaching of secondary school mathematics. (Indiana University, 1958.) Dissertation Abstracts, 1959, 19, 2033.

- Shipp, D. E. An experimental study of achievement in arithmetic and the time allotted to development of meanings and individual pupil practice. (Louisiana State University, 1958.) Dissertation Abstracts, 1958, 19, 492.
- Skager, R. W. Student entry skills and the evaluation of instructional programs: A case study. Paper presented at the Annual Meeting of the American Educational Research Association, Los Angeles, California, February 1969. (ERIC: ED 029 364)
- SMMSG. Report of a conference on mathematics education in the inner city schools. Stanford, California: Stanford University School Mathematics Study Group, March 1970. (ERIC: ED 083 008)
- Smith, D. S. The relationship between classroom means of students' perceptions of teachers as related to classroom racial composition and grade level, and the effects on classroom means of academic growth. (Western Michigan University, 1974.) Dissertation Abstracts International, 1974, 35A, 3309.
- Stallings, J. Relationships between classroom instructional practices and child development. Menlo Park, California: Stanford Research Institute, March 1975. (ERIC: ED 110 200)
- Standards of quality and objectives for public schools in Virginia 1976-78. Richmond, Virginia: Department of Education, 1976.
- Stickney, B. D. Compensatory education and pupil achievement. (University of Massachusetts, 1976.) Dissertation Abstracts International, 1976, 37A, 2088.
- Stiglmeier, L. M. Teachers' judgments of pupils' dependence/self-reliance characteristics mode of instruction and their relationship to achievement. (State University of New York at Albany, 1972.) Dissertation Abstracts International, 1973, 34A, 1008.
- Stillwell, L. J. An investigation of the interrelationships among global self concept, role self concept and achievement. (Western Reserve University, 1965.) Dissertation Abstracts, 1966, 27A, 682.
- Stilwell, M. E. The development and analysis of a category system for systematic observation of teacher pupil interaction during geometry problem-solving activity. (Cornell University, 1967.) Dissertation Abstracts, 1968, 28A, 3083.
- Stodolsky, S., & Lesser, G. Learning patterns in the disadvantaged. Harvard Educational Review, 1967, 37, 546-593.

- Strickmeier, H. B., Jr. An analysis of verbal teaching behaviors in seventh grade mathematics classes grouped by ability. (University of Texas at Austin, 1970.) Dissertation Abstracts International, 1971, 31A, 3428.
- Summers, A. A., & Wolfe, B. L. Which school resources help learning? Efficiency and equity in Philadelphia public schools. In Business Review. Philadelphia: Federal Reserve Bank of Philadelphia, 1975.
- Suydam, M. N. A review of research on secondary school mathematics. Columbus, Ohio: ERIC Information Analysis Center for Science, Mathematics, and Environmental Education, 1972. (ERIC: ED 065 313)
- Suydam, M. N. Research on some key non-cognitive variables in mathematics education. In Schriftenreihe des IDM. Bielefeld, Germany: Institut für Didaktik der Mathematik, Universität Beilefeld, 1975, pp. 105-135. (ERIC: 133 183)
- Suydam, M. N. Electronic hand calculators: The implications for pre-college education (Final Report). Washington: National Science Foundation, February 1976. (ERIC: ED 127 205)
- Suydam, M. N. (Ed.) Developing computational skills (1978 Yearbook). Reston, Virginia: National Council of Teachers of Mathematic, 1978.
- Suydam, M. N. State-of-the-art review on calculators: Their use in education. Columbus, Ohio: Calculator Information Center, April 1978.
- Suydam, M. N., & Dessart, D. J. Classroom ideas from research on computational skills. Reston, Virginia: National Council of Teachers of Mathematics, 1976.
- Suydam, M. N., & Higgins, J. L. Review and synthesis of studies of activity-based approaches to mathematics teaching. NIE Contract No. 400-75-0063. Final Report, September 1976. (Also available from ERIC Information Analysis Center for Science, Mathematics, and Environmental Education.)
- Suydam, M. N., & Osborne, A. The status of pre-college science, mathematics, and social science education: 1955-1975 (Volume II: Mathematics Education). National NSF Foundation, Contract NSF-C7620627, 1977.
- Suydam, M. N., & Riedesel, C. A. Interpretive study of research and development in elementary school mathematics (Volume 3: Developmental Projects). University Park: The Pennsylvania State University, 1969. (ERIC: ED 030 018)
- Suydam, M. N., & Weaver J. F. Using research: A key to elementary school mathematics. University Park: The Pennsylvania State University, 1970.

- Suydam, M. N., & Weaver J. F. Mathematics and the 'disadvantaged.' Columbus, Ohio: ERIC Information Analysis Center for Science, Mathematics, and Environmental Education, March 1971. (ERIC: ED 049 063)
- Suydam, M. N., & Weaver, J. F. Using research: A key to elementary school mathematics. Columbus, Ohio: ERIC Information Analysis Center for Science, Mathematics, and Environmental Education, 1975. (ERIC: ED 120 013)
- Tanner, R. T. Discovery as an object of research. School Science and Mathematics, 1969, 69, 647-655.
- Thomas, Gregory P. Field impact evaluation. Monmouth, Oregon: Teaching Research, December 1975.
- Those dropping test scores--Experts grope for the reasons. U.S. News & World Report, Nov.24, 1975, 34.
- Trafton, P. R., & Morgan, V. R. Mathematics in urban schools. Arithmetic Teacher, December 1978, 26, 3-4.
- Unkel, E. A study of the interaction of socioeconomic groups and sex factors with the discrepancy between anticipated achievement and actual achievement in elementary school mathematics. Arithmetic Teacher, December 1966, 13, 662-670.
- Visonhaler, J. F., & Boss, R. K. A summary of ten major studies on CAI drill and practice. Educational Technology, July, 1972, 12, 29-32.
- Weaver, J. F., & Suydam, M. N. Meaningful instruction in mathematic education. Columbus, Ohio: ERIC Information Analysis Center for Science, Mathematics, and Environmental Education, June 1972. (ERIC: ED 068 329)
- Weimer, R. C. A critical analysis of the discovery versus expository research studies investigating retention or transfer within the areas of science, mathematics, vocational education, language, and geography from 1908 to the present. (University of Illinois at Urbana-Champaign, 1974.) Dissertation Abstracts International, 1975, 35A, 7185-7186.
- Wellisch, J. B. et al. An in-depth study of Emergency School Aid Act (ESAA) schools: 1974-1975. Santa Monica, California: System Development Corporation, July 1976. (ERIC: ED 133 361)
- Zahn, K. G. The optimum ratio of class time to be allotted to developmental activities and to individual practice in teaching arithmetic. (University of Colorado, 1965.) Dissertation Abstracts, 1966, 26, 6459.

Zander, B. J. Junior high students view themselves as learners: A comparison among eighth grade students. (University of Minnesota, 1973.) Dissertation Abstracts International, 1973, 34A, 2254-2255.

Zoll, E. J. Research in programmed instruction in mathematics. Mathematics Teacher, 1969, 62, 103-110.

ERIC: ED 132 176. Impact of educational innovation on student performance: Project methods and findings for three cohorts. (Project LONGSTEP Final Report: Volume I. Executive Summary). Palo Alto, California: American Institutes for Research in the Behavioral Sciences, April 1976.

ERIC: ED 132 178. Impact of educational innovation on student performance: Project methods and findings for three cohorts. (Project LONGSTEP Final Report: Volume I. Appendices). Palo Alto, California: American Institutes for Research in the Behavioral Sciences, April 1976.

ERIC: ED 132 193. Impact of educational innovation on student performance: Overall findings for reading and arithmetic. (Project LONGSTEP Final Report: Volume I. Supplement. Supporting Appendices). Palo Alto, California: American Institutes for Research in the Behavioral Sciences, September 1976.

OTHER TITLES IN THE SERIES

Volume 1

Compensatory Education: A
Congressional Perspective
Christopher T. Cross, the U.S.
House of Representatives

Volume 2

Effective Teacher-Support Systems
to Improve Instructional
Conditions in Schools
Joseph D. Liggins, Houston
Independent School District

Volume 3

Assessment That Respects
Complexity in Individuals and
Programs
Judah L. Schwartz, Massachusetts
Institute of Technology

Volume 4

Achievement Tests in Urban Schools
Eva Baker, University of
California, Los Angeles

Volume 5

Instructional Ingredients for the
Development of Beginning Reading
Competence
Isabel L. Beck, University of
Pittsburgh

Volume 6

A Discussion of the Literature
and Issues Related to Effective
Schooling
Ronald Edmonds, Harvard
University and New York
City Public Schools

Volume 7

Summer Drop-Off and the
Effectiveness of Compensatory
Instruction
Paul T. Hill, The Rand Corporation

Volume 8

The Educational Promise of
Cultural Pluralism
Vera John Steiner and Larry
Smith, The University of New
Mexico

Volume 9

New Perspectives on School
District Research and Evaluation
Catherine Lyon, University of
California, Los Angeles

Volume 10

Effective Teaching and Learning
in Urban Schools
Jane A. Stallings and Shirley W.
Hentzell, SRI International

Volume 11

Mathematics and the Urban Child:
A Review of Research
Marilyn N. Suydam, The Ohio State
University

Volume 12

Basic Skills in Urban Schools: A
View from the Bilingual Classroom
Concepcion M. Valadez, University
of California, Los Angeles

Volume 13

School-Community Relations and
Student Achievement
John E. Ingram, Jr. and Lois I.
Bartels, St. Louis Public
Schools and CEMREL, Inc.

Volume 14

What Do We Know About Teaching
and Learning in Urban Schools: A
Summary of the Conference Findings
Edmund W. Gordon, Columbia
University